PART – A

UNIT 1:

Basic Concepts: Practical sources, Source transformations, Network reduction using Star – Delta transformation, Loop and node analysis With linearly dependent and independent sources for DC and AC networks, Concepts of super node and super mesh

7 Hours

UNIT 2:

Network Topology: Graph of a network, Concept of tree and co-tree, incidence matrix, tie-set, tie-set and cut-set schedules, Formulation of equilibrium equations in matrix form, Solution of resistive networks, Principle of duality**.**

7 Hours

UNIT 3:

Network Theorems – 1: Superposition, Reciprocity and Millman's theorems

6 Hours

UNIT 4:

Network Theorems - II:

Thevinin's and Norton's theorems; Maximum Power transfer theorem

6 Hours

PART – B

UNIT 5: Resonant Circuits: Series and parallel resonance, frequency-response of series and Parallel circuits, Q –factor, Bandwidth.

6Hours

UNIT 6:

Transient behavior and initial conditions: Behavior of circuit elements under switching condition and their Representation, evaluation of initial and final conditions in RL, RC and RLC circuits for AC and DC excitations.

7 Hours

UNIT 7:

Laplace Transformation & Applications : Solution of networks, step, ramp and impulse responses, waveform Synthesis

7 Hours

UNIT 8:

Two port network parameters: Definition of z, y, h and transmission parameters, modeling with these parameters, relationship between parameters sets

6 Hours

TEXT BOOKS:

1. **"Network Analysis",** M. E. Van Valkenburg, PHI / Pearson Education, 3rd Edition. Reprint 2002.
2. **"Networks and systems",** Roy Choudhury, 2nd edition, 2006 re-print, New Age International Publications.

REFERENCE BOOKS:

1. **, "Engineering Circuit Analysis",** Hayt, Kemmerly and DurbinTMH 6th Edition, 2002
2. **"Network analysis and Synthesis",** Franklin F. Kuo, Wiley International Edition,
3. **"Analysis of Linear Systems"**, David K. Cheng, Narosa Publishing House, 11th reprint, 2002
4. **"Circuits"**, Bruce Carlson, Thomson Learning, 2000. Reprint 2002

Question Paper Pattern: Student should answer FIVE full questions out of 8 questions to be set each carrying 20 marks, **selecting at least TWO questions from each part**.

Coverage in the Texts:

UNIT 1: Text 2: 1.6, 2.3, 2.4 (Also refer **R1:**2.4, 4.1 to 4.6; 5.3, 5.6; 10.9 This book gives concepts of super node and super mesh)

UNIT 2: Text 2: 3.1 to 3.11

UNIT 3 and UNIT 4: Text 2 – 7.1 to 7.7

UNIT 5: Text 2 – 8.1 to 8.3

UNIT 6: Text 1 – Chapter 5;

UNIT 7: Text 1 – 7.4 to 7.7; 8.1 to 8.5

UNIT 8: Text 1 – 11.1 to 11.

INDEX SHEET

Unit: 1 : Basic Concepts Hrs: 07

Syllabus of unit 1:

Practical sources, Source transformations, Network reduction using Star – Delta transformation, Loop and node analysis With linearly dependent and independent sources for DC and AC networks Concepts of super node and super mesh.

Recommended readings:

1. "Network Analysis", M. E. Van Valkenburg, PHI / Pearson Education

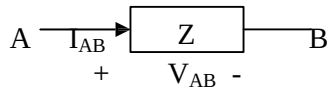
2. "Networks and systems", Roy Choudhury, 2 edition, New Age International Publications .

3. "Network theory ", Ganesh Rao.

4. "Network analysis" , Roy Choudry.

BASIC LAWS:

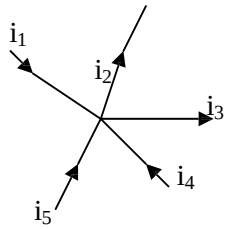
1. OHMS LAW V=IZ



I_{AB}-Current from A to B V_{AB}=Voltage of A w.r.t B

2. KCL

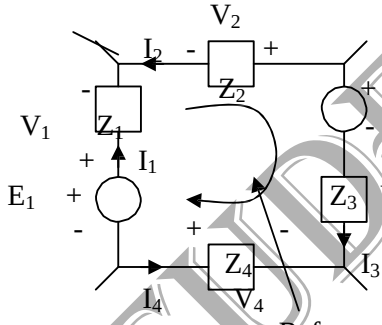
$i_1+i_4+i_5=i_2+i_3$



i=0 algebraic sum

or $i_{in}= i_{iout}$ ($I_{in}=-I_{out}$)

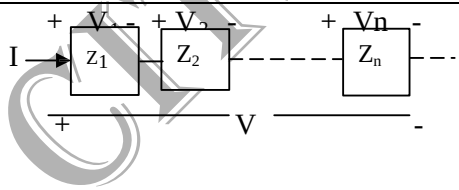
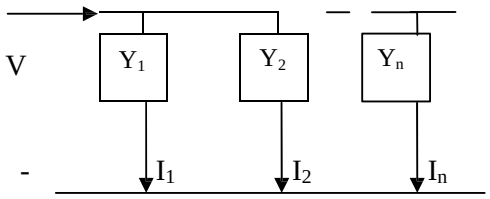
3. KVL v=0 algebraic sum



$v_{rise}= v_{drop}$ ($V_{rise}= V_{drop}$)

$E_1-E_2=V_1-V_2+V_3-V_4=I_1Z_1-I_2Z_2+I_3Z_3-I_4Z_4$

CONNECTIONS

SERIES	PARELLEL
	
$Z = \sum_{1}^{n} Z_K = Z_1 + Z_2 + Z_3 - - - Z_n$	$Y = \sum_{1}^{n} Y_K = Y_1 + Y_2 + Y_3 - - - Y_n$
Voltage Division $V_i=(Z_i/Z)V$	Current Division $I_I=(Y_i/Y)I$

$I=V/Z=V_1/Z_1=V_2/Z_2=$----------	$V=I/Y=I_1/Y_1=I_2/Y_2=$----------

Problems

1.Calculate the voltages V_{12}, V_{23}, V_{34} in the network shown in Fig, if $V_a=17.32+j10$ $V_b=30\angle 80^0$ V and $V_c=15\angle -100$V

with Calculator in complex and degree mode

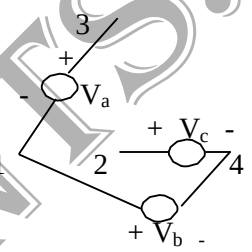
$V_{12} = -V_c + V_b$

$= (0-15\angle -100 + 30\angle 80) = 45\angle 80^0$ V *

$V_{23} = V_a - V_b + V_c = V_a - V_{12}$

$= 17.32+10i - 45\angle 80^0 = 35.61\angle -74.52^0$

$V_{34} = V_b - V_a = 30\angle 80 - 17.32-10i = 23\angle 121.78^0$



2.How is current of 10A shared by 3 impedances $Z_1=2-J5$[2 $Z_2 = 6.708\angle 26.56$ and $Z_3 = 3 + J4$ all connected in parallel

Ans:

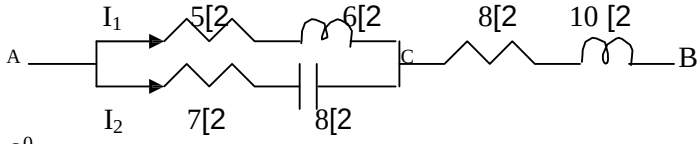
$Z = Y^{-1} = ((2-5i)^{-1} + (6.708\angle 26.56)^{-1} + (3+4i)^{-1} = 3.06\angle 9.55^0$

$V=1Z = 30.6\angle 9.55^0$ $I_1 = V/Z_1=(30.6\angle 9.55^0)$ —: $(2-5i) = 5.68\ 77.75^0$

$I_2 = \frac{V}{Z_2} = (30.6\angle 9.55^0) \div (6.708\angle 26.56) = 4.56\angle -17^0$

$I_3 = \frac{V}{Z_2} = (30.6\angle 9.55^0) \div (3+4i) = 6.12\angle -43.6^0$

3. In the circuit determine what voltage must be applied across AB in order that a current of 10 A may flow in the capacitor



$V_{AC} = (7-8i)(10) = 106.3\angle -48.8^0$

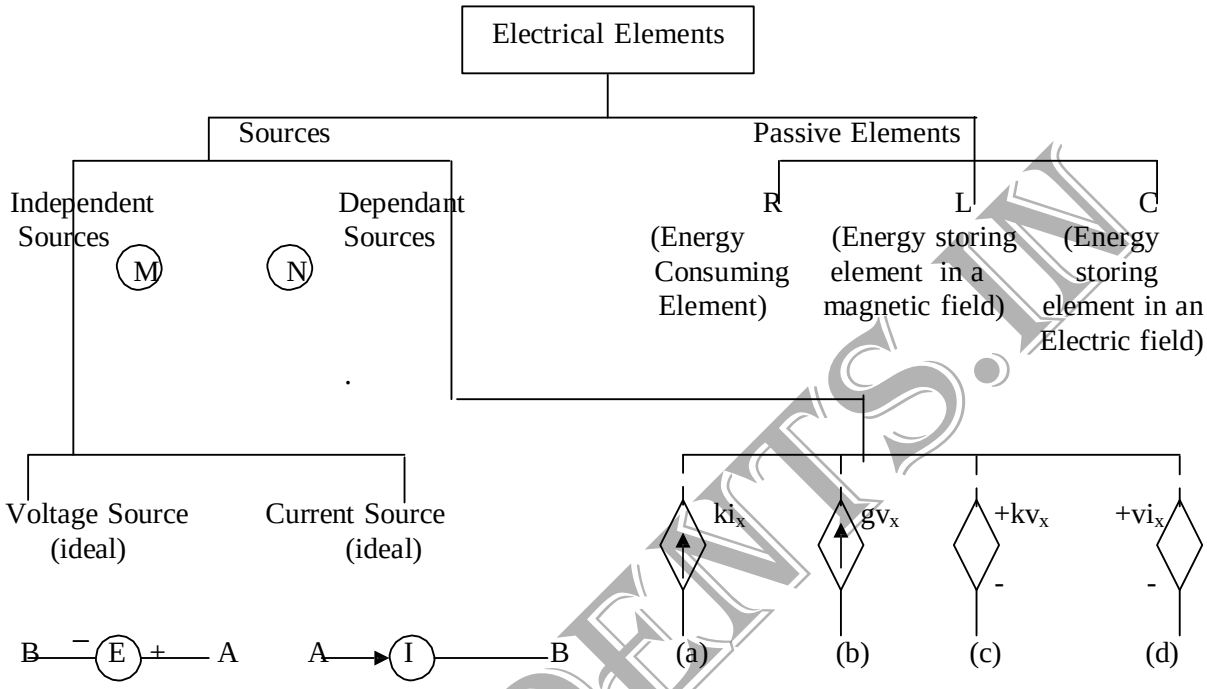
$I_1 = \frac{V_{AC}}{5+6i} = 13.61\angle -99^0$

$I = I_1 + I_2 = 10\angle 0^0 + 13.61\angle -99^0 = 15.576\angle -59.66^0$

$V = V1+V2 = 106.3\angle -48.8 + (15.576\angle -59.66)(8+10i) = 289\angle -22^0$

Practical sources:

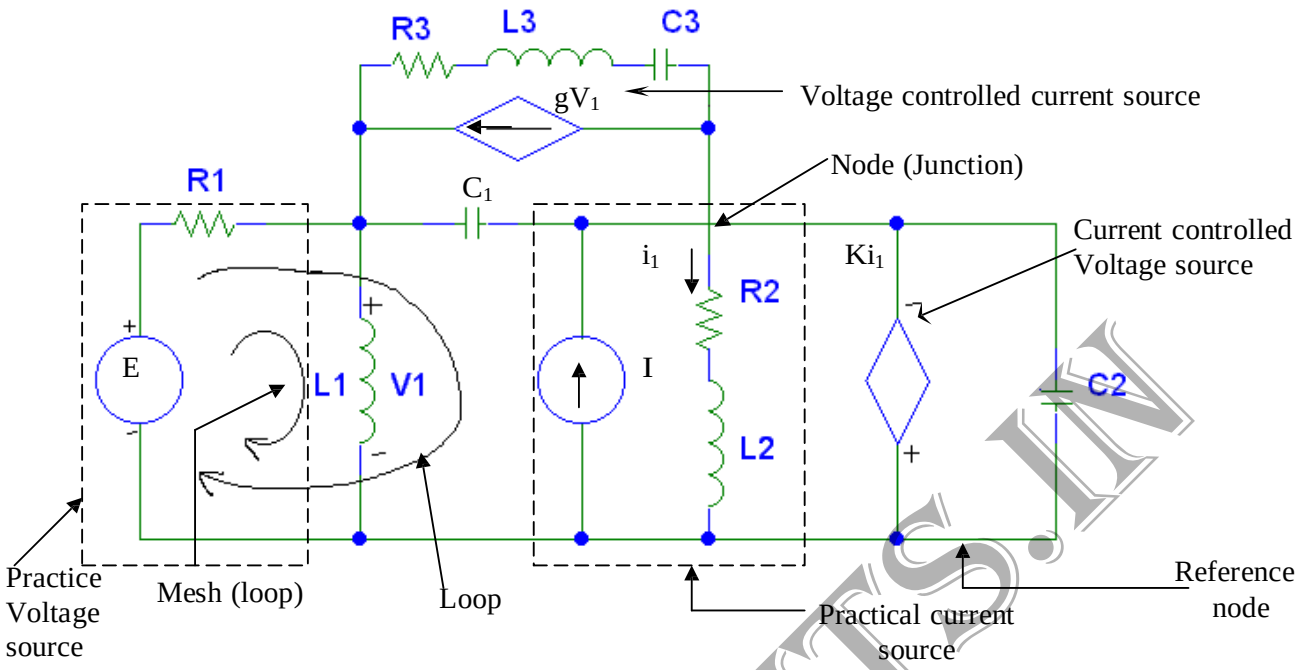
Network is a system with interconnected electrical elements. Network and circuit are the same. The only difference being a circuit shall contain at least one closed path.



(a)Current controlled current source
(b) Voltage controlled current source
(c) Voltage controlled voltage source
(d) Current controlled voltage source

M
(Value of source Quantity is not affected in anyway by activities in the reminder of the circuit.)

N
(Source quantity is determined by a voltage or current existing at some other Location in the circuit)
These appear in the equivalent models for many electronic devices like transistors, OPAMPS and integrated circuits.



TYPES OF NETWORKS

Linear and Nonlinear Networks:

A network is linear if the principle of superposition holds i.e if e1(t), r1 (t) and e2(t), r2 (t) are excitation and response pairs then if excitation is e1 (t) + e2 (t) then the response is r1 (t) + r2(t).

The network not satisfying this condition is nonlinear

Ex:- Linear – Resistors, Inductors, Capacitors.

Nonlinear – Semiconductors devices like transistors, saturated iron core inductor, capacitance of a p-n function.

Passive and active Networks:

A Linear network is passive if (i) the energy delivered to the network is nonnegative for any excitation. (ii) no voltages and currents appear between any two terminals before any excitation is applied.

Example:- R,L and C.

Active network:- Networks containing devices having internal energy –Generators, amplifiers and oscillators.

Unilateral & Bilateral:

The circuit, in which voltage current relationship remains unaltered with the reversal of polarities of the source, is said to be bilateral.

Ex:- R, L & C

If V-I relationships are different with the reversal of polarities of the source, the circuit is said to be unilateral. Ex:- semiconductor diodes.

Lumped & Distributed:

Elements of a circuit, which are separated physically, are known as lumped elements.

Ex:- L & C.

Elements, which are not separable for analytical purposes, are known as distributed elements.

Ex:- transmission lines having R, L, C all along their length.

In the former care Kirchhoff's laws hold good but in the latter case Maxwell's laws are required for rigorous solution.

Reciprocal:

A network is said to be reciprocal if when the locations of excitation and response are interchanged, the relationship between them remains the same.

Source Transformation:

In network analysis it may be required to transform a practical voltage source into its equivalent practical current source and vice versa . These are done as explained below

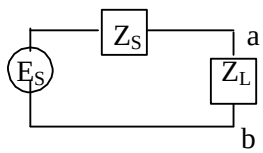


fig 1

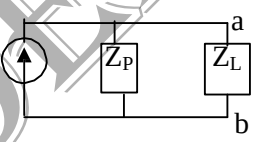


fig 2

Consider a voltage source and a current source as shown in Figure 1 and 2. For the same load Z_L across the terminals a & b in both the circuits, the currents are

$I_L = \frac{E_S}{Z_s + Z_L}$ in fig 1 and $I_L = \frac{I_S . Z_P}{Z_p + Z_L}$ in fig 2

For equivalence $\frac{E_S}{Z_S + Z_L} = I_S . \frac{Z_P}{Z_P + Z_L}$

Therefore $E_S = I_S Z_P$ and $Z_S = Z_P$

Therefore

$$I_S = \frac{E_S}{Z_P} = \frac{E_S}{Z_S}$$

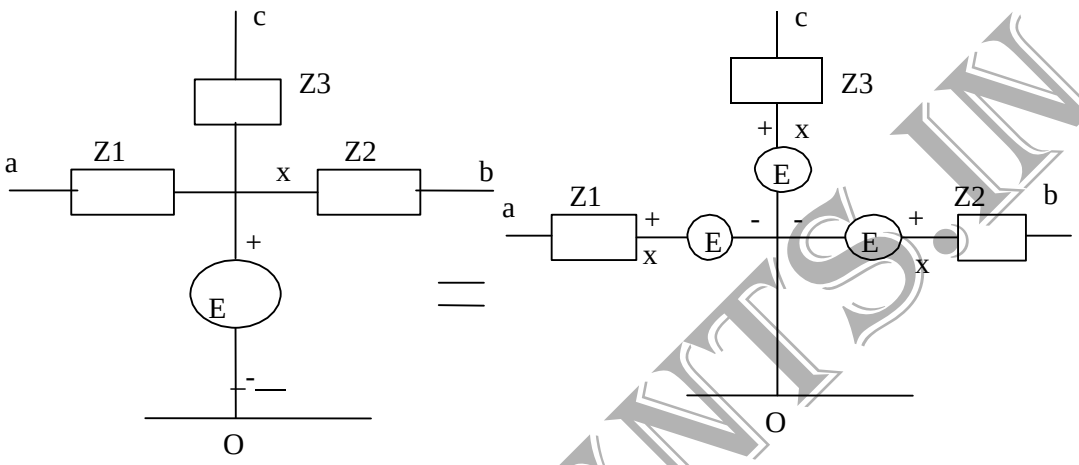
Transformation from a practical voltage source to a practical current source eliminates a node.
Transformation from a practical current source to a current source eliminates a mesh.

A practical current source is in parallel with an impedance Zp is equivalent to a voltage source Es=Is Zp in series with Zp.

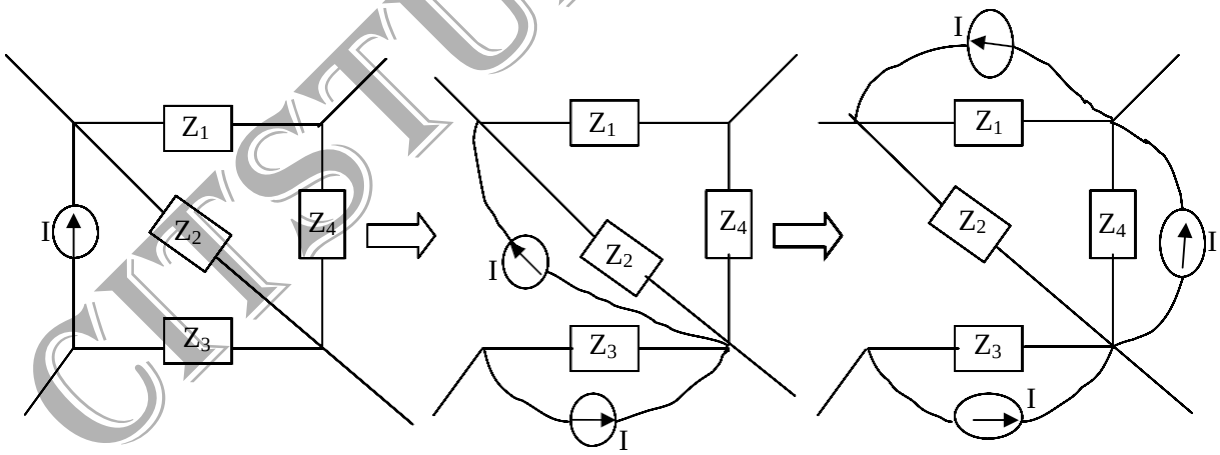
A practical voltage source Es in series with a impedance Zs is equivalent to a current source Es/Zs in parallel with Zs.

SOURCE SHIFTING:

Source shifting is occasionally used to simplify a network. This situation arises because of the fact than an ideal voltage source cannot be replaced by a current source. Like wise ideal current source cannot be replaced by a voltage source. But such a source transformation is still possible if the following techniques are fallowed.

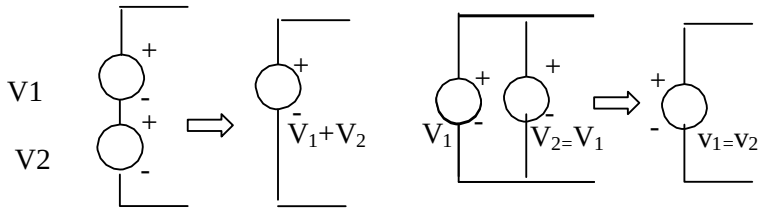


(a) E shift operation

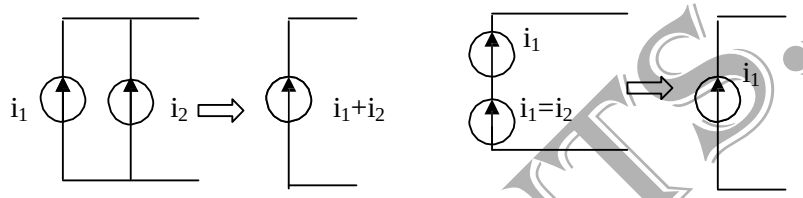


(b) I shift operation

Sources with equivalent terminal characteristics

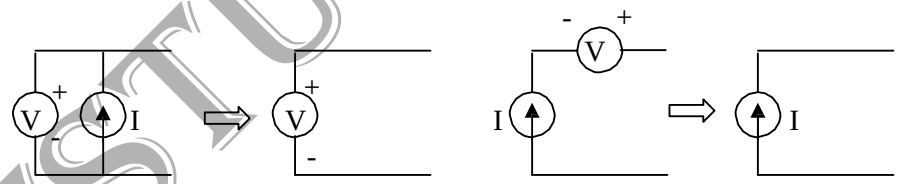


(i)Series voltage sources (ii) Parallel voltage sources(ideal)



(iii) Parallel current sources (iv)Series current sources(ideal)

(v)Voltage source with parallel Z (vi)Current source with series Z



(vii) V and I in Parallel (viii) V and I in Series

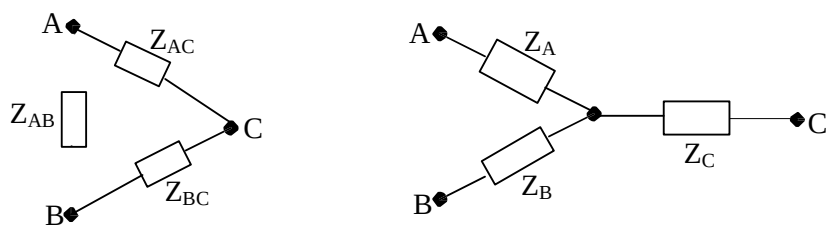
Delta-star transformation:

A set of star connected (Y o:r T) immittances can be replaced by an equivalent set of mesh (Δ or 7t) connected immittances or vice versa. Such a transformation is often necessary to simplify passive networks, thus avoiding the need for any mesh or nodal analysis.

For equivalence, the immittance measured between any two terminals under specified conditions must be the same in either case.

Δ to Y transformation:

Consider three Δ-connected impedances Z_{AB}, Z_{BC} and Z_{CA} across terminals A, B and C. It is required to replace these by an equivalent set Z_A, Z_B and Z_C connected in star.



In Δ, impedance measured between A and B with C open is

$$\frac{Z_{AB}(Z_{BC} + Z_{CA})}{Z_{AB} + Z_{BC} + Z_{CA}}$$

With C open, in Y, impedance measured between A and B is $Z_A + Z_B$.

For equivalence $Z_A + Z_B = \dfrac{Z_{AB}(Z_{BC} + Z_{CA})}{Z_{AB} + Z_{BC} + Z_{CA}}$ ----------(1)

Similarly for impedance measured between B and C with A open

$$Z_B + Z_C = \frac{Z_{BC}(Z_{CA} + Z_{AB})}{Z_{AB} + Z_{BC} + Z_{CA}} \quad \text{----------(2)}$$

For impedance measured between C and A with B open

$$Z_C + Z_A = \frac{Z_{CA}(Z_{AB} + Z_{BC})}{Z_{AB} + Z_{BC} + Z_{CA}} \quad \text{----------(3)}$$

Adding (1), (2) and (3)

$$2(Z_A + Z_B + Z_C) = \frac{2(Z_{AB}Z_{BC} + Z_{BC}Z_{CA} + Z_{CA}Z_{AB})}{Z_{AB} + Z_{BC} + Z_{CA}}$$

$$Z_A = \frac{(Z_{AB}Z_{BC} + Z_{BC}Z_{CA} + Z_{CA}Z_{AB})}{Z_{AB} + Z_{BC} + Z_{CA}} - (Z_B + Z_C)$$

Substituting for $Z_B + Z_C$ from (2)

$$Z_A = \frac{Z_{CA}Z_{AB}}{Z_{AB} + Z_B + Z_{CA}} = \frac{Z_{CA}Z_{AB}}{Z_{AB}}$$

Similarly by symmetry $Z_B = \dfrac{Z_{AB}Z_{BC}}{Z_{AB}}$

$$Z_C = \frac{Z_{BC}Z_{CA}}{Z_{AB}}$$

If $Z_{AB} = Z_{BC} = Z_{CA} = Z_{!J}$ then $Z_A = Z_B = Z_C = Z_Y = \dfrac{Z_{!J}}{3}$.

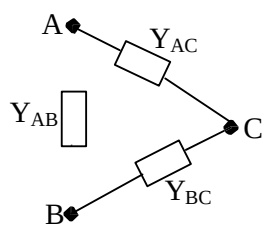
Y to A transformation:

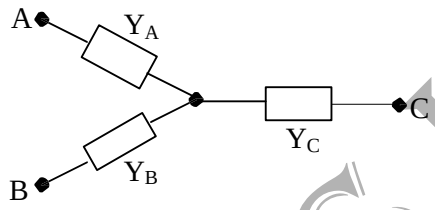
Consider three Y connected admittance Y_a, Y_b and Y_c across the terminals A, B and C. It is required to replace them by a set of equivalent !J admittances Y_{ab}, Y_{bc} and Y_{ca}.

Admittance measured between A and B with B & C shorted

In Y $\dfrac{Y_A (Y_B + Y_C)}{Y_A + Y_B + Y_C}$

In !J $Y_{AB} + Y_{CA}$





For equivalence $Y_{AB} + Y_{CA} = \dfrac{Y_A (Y_B + Y_C)}{Y_A + Y_B + Y_C}$ ----------(1)

Admittance between B and C with C & A shorted

$Y_{BC} + Y_{AB} = \dfrac{Y_B (Y_C + Y_A)}{Y_A + Y_B + Y_C}$ ----------(2)

Admittance between C and A with A & B shorted

$Y_{CA} + Y_{BC} = \dfrac{Y_C (Y_A + Y_B)}{Y_A + Y_B + Y_C}$ ----------(3)

Adding (1), (2) and (3) $Y_{AB} + Y_{BC} + Y_{CA} = \dfrac{Y_A Y_B + Y_B Y_C + Y_C Y_A}{Y_A + Y_B + Y_C}$

$Y_{AB} = \dfrac{\text{?}\, Y_A Y_B}{\text{?}\, Y_A} - (Y_{BC} + Y_{CA})$

substituting from (3)

$= \dfrac{Y_A Y_B}{Y_A + Y_B + Y_C}$: $Y_{BC} = \dfrac{Y_B Y_C}{Y_A + Y_B + Y_C}$: $Y_{CA} = \dfrac{Y_A Y_B}{Y_A + Y_B + Y_C}$

In terms of impedances,

$Z_{AB} = \dfrac{Y_A + Y_B + Y_C}{Y_A Y_B} = \dfrac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_C}$

Similarly $Z_{BC} = \dfrac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_A}$

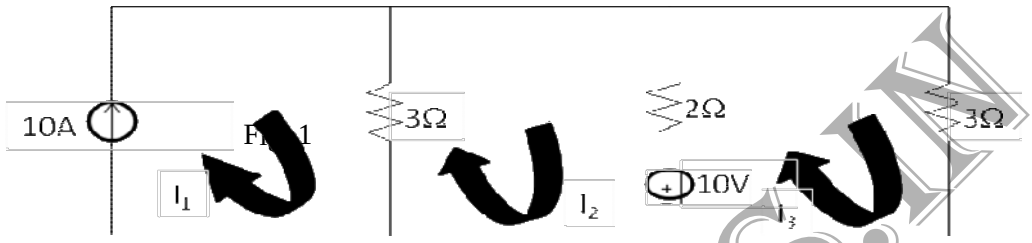
$Z_{CA} = \dfrac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_B}$

If $Z_A = Z_B = Z_C = Z_Y$ then $Z_{AB} = Z_{BC} = Z_{CA} = Z_{!J} = 3Z_Y$.

Assignment questions:

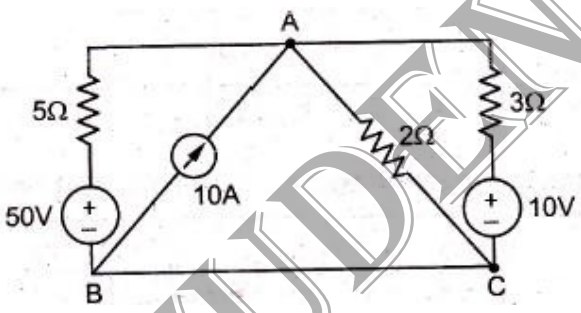
1) Distinguish the following with suitable examples.
   i) Linear and non-linear elements.
   ii) Unilateral and trilateral elements.
   iii) Independent and dependent sources.

2) Write the mesh equation for the circuit shown in Fig. 1 and determine mesh currents using mesh analysis.



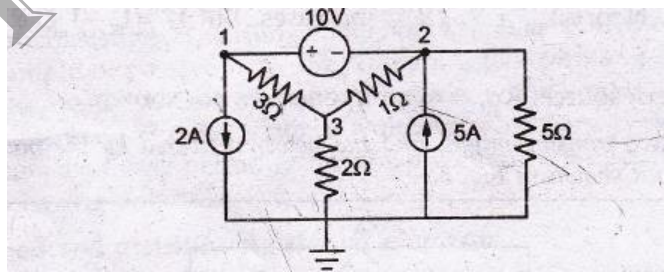
3) Establish star – delta relationship suitably

4) Using source transformation, find the power delivered by the'50 V voltage source in the circuit shown in Fig.



---

5) Find the power delivered by the 5A current source in the circuit shown in Fig.4 by using the nodal method.



6) For the network shown in fig. 2, determine the voltage V using source shift and / or source transformation techniques only. Then verify by node equations

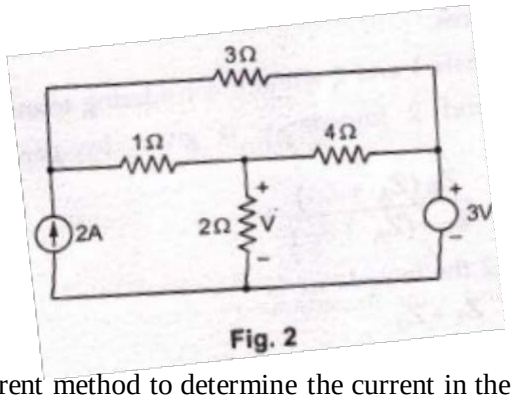


Fig. 2

7) Use mesh current method to determine the current in the capacitor of 6Ω of the bridge network shown in fig. 5.

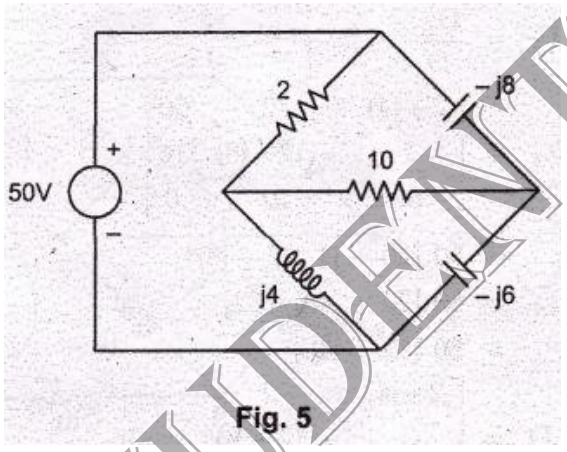


Fig. 5

8) Use node equations to determine what value of 'E' will cause V_x to be zero for the network shown in fig. 6.

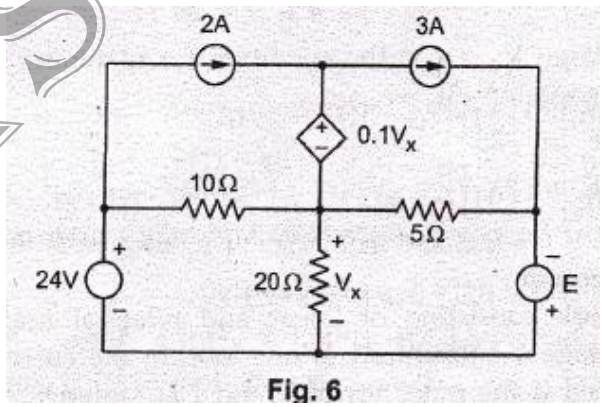


Fig. 6

9) Obtain the delta connected equipment of the network shown in fig. 1.

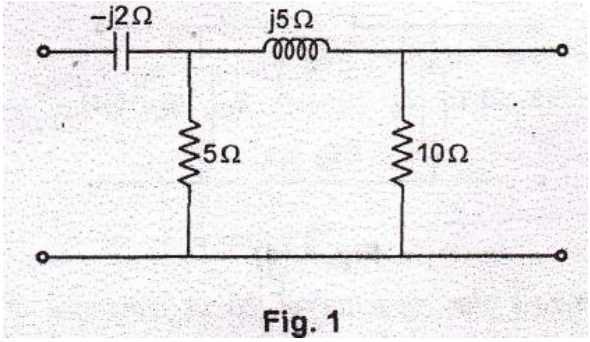


Fig. 1

10) c) Find the voltage across the capacitor of 20 Ω reactance of the network shown in fig. 2, by reducing the network to contain one source only, by source transformation techniques.

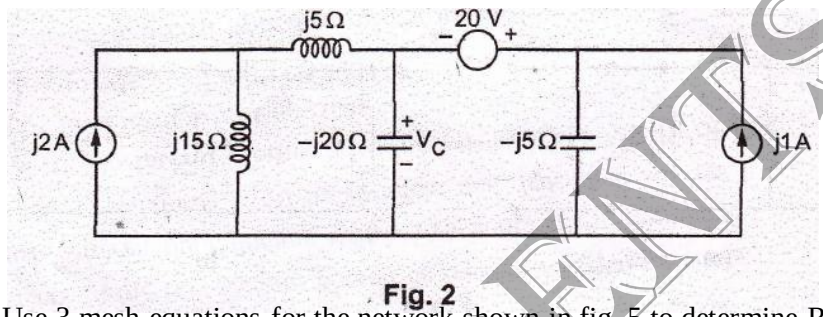
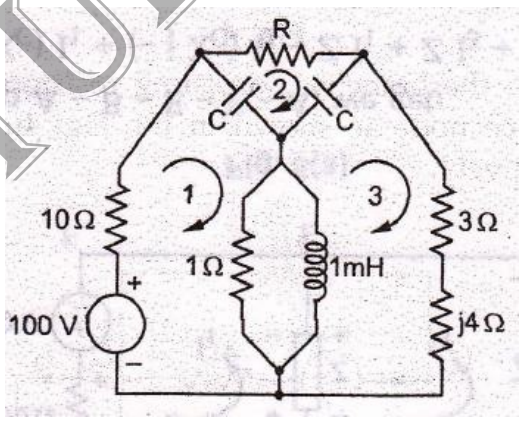


Fig. 2

11) c) Use 3 mesh equations for the network shown in fig. 5 to determine R and C. such that the current in 3+j4Ω is zero . Take ω = 50 rad/sec.



Unit: 2 Network Topology : Hrs: 07

Syllabus of unit :

Graph of a network, Concept of tree and co-tree, incidence matrix, tie-set, tie-set and cut-set schedules, Formulation of equilibrium equations in matrix form, Solution of resistive networks,
Principle of duality

Recommended readings:

1. "Network Analysis", M. E. Van Valkenburg, PHI / Pearson Education

2. "Networks and systems", Roy Choudhury, 2 edition, New Age International Publications .

3. "Network theory ", Ganesh Rao.

4. "Network analysis" , Roy Choudry.

Net work Topology

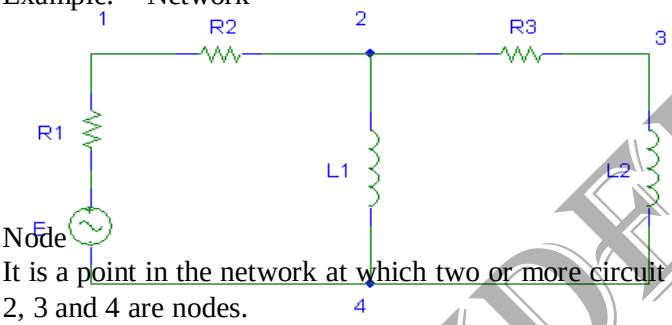
Definition:

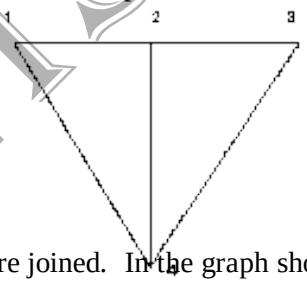
The term circuit topology refers to the science of placement of elements and is a study of the geometric configurations.
"Circuit topology is the study of geometric properties of a circuit useful for describing the circuit behavior"

Terms used in Topology:
The following terms are often used in network topology

Graph:
In the given network if all the branches are represented by line segments then the resulting figure is called the graph of a network (or linear graph). The internal impedance of an ideal voltage source is zero and hence it is replaced by a short circuit and that of an ideal current source is infinity and hence it is represented by an open circuit in the graph.

Example: Network Graph



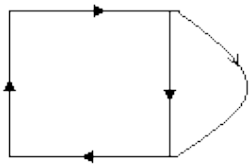


Node
It is a point in the network at which two or more circuit elements are joined. In the graph shown 1, 2, 3 and 4 are nodes.

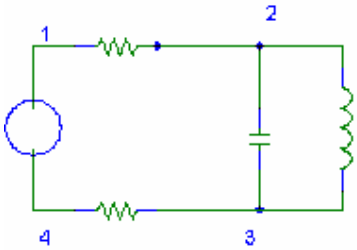
Branch (or Twig):

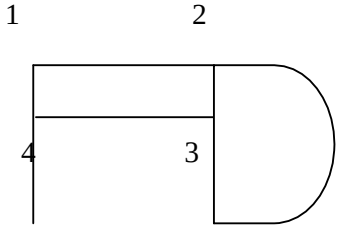
It is a path directly joining two nodes. There may be several parallel paths between two nodes.

Oriented Graph
If directions of currents are marked in all the branches of a graph then it is called an oriented (or directed) graph .



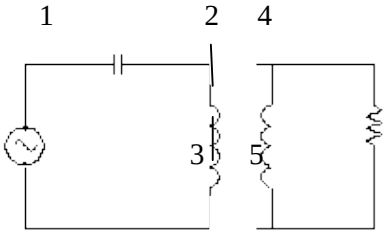
Connected graph
A network graph is connected if there is a path between any two nodes .In our further discussion,let us assume that the graph is connected. Since, if it is not connected each disjoint part may be analysed separately as a connected graph.

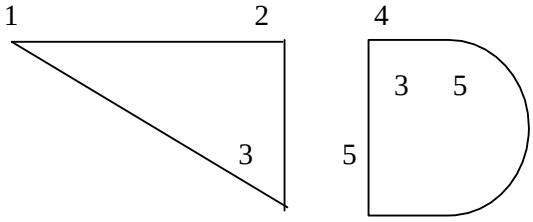




Unconnected graph

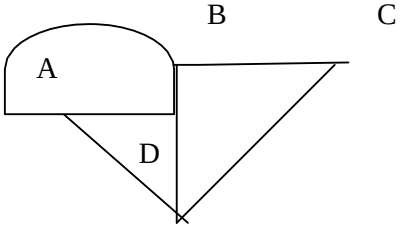
If there is no path between any two nodes,then the graph is called an unconnected graph.

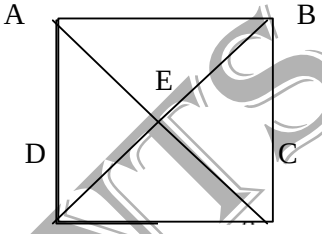




Planar graph

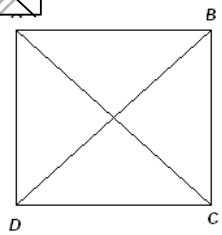
A planar graph is a graph drawn on a two dimensional plane so that no two branches intersect at a point which is not a node.





Non – planar graph

A graph on a two – dimensional plane such that two or more branches intersect at a point other than node on a graph.



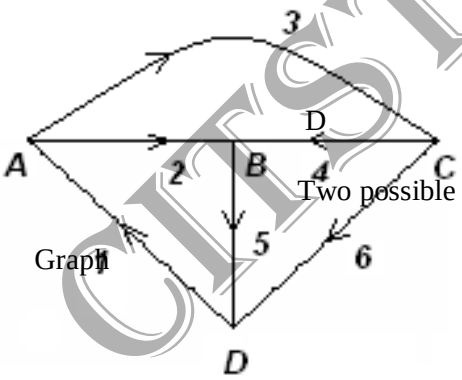
Tree of a graph

Tree is a set of branches with all nodes not forming any loop or closed path.

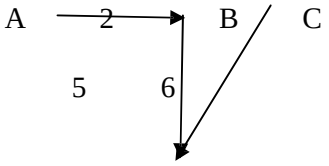
(*) Contains all the nodes of the given network or all the nodes of the graph

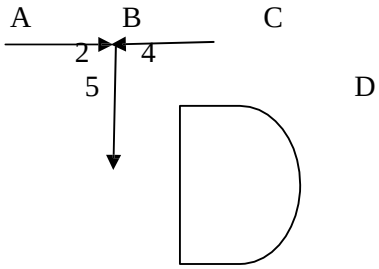
(*) No closed path

(*) Number of branches in a tree = n-1 , where n=number of nodes



Graph





Two possible trees

Co- tree

A Co- tree is a set of branches which are removed so as to form a tree or in other words, a co- tree is a set of branches which when added to the tree gives the complete graph. Each branch so removed is called a link.

Number of links = l = b – (n-1) where b = Total number of branches

n = Number of nodes

Incidence Matrix

Incidence matrix is a matrix representation to show which branches are connected to which nodes and what is their orientation in a given graph

(*) The rows of the matrix represent the nodes and the columns represents the branches of the graph.

(*) The elements of the incidence matrix will be +1, -1 or zero

(*) If a branch is connected to a node and its orientation is away from the node the corresponding element is marked +1

(*) If a branch is connected to a node and its orientation is towards the node then the corresponding element is marked – 1

(*) If a branch is not connected to a given node then the corresponding element is marked zero.

Incidence Matrix

Complete Incidence matrix | Reduced incidence matrix

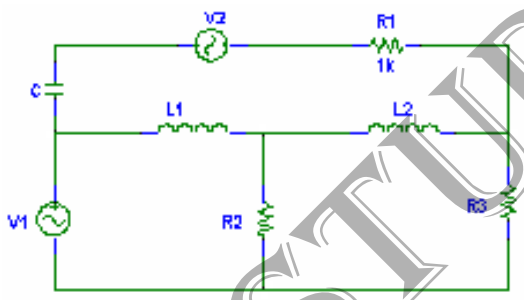
(i)Complete incidence matrix:

An incidence matrix in which the summation of elements in any column in zero is called a complete incidence matrix.

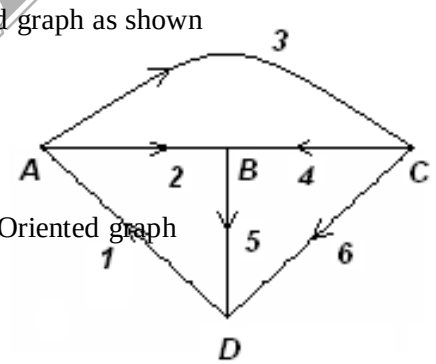
(ii)Reduced incidence matrix:

The reduced incidence matrix is obtained from a complete incidence matrix by eliminating a row. Hence the summation of elements in any column is not zero.

Example 1: Consider the following network and the oriented graph as shown



Network



Oriented graph

(*) There are four nodes A, B, C and D and six branches 1, 2, 3, 4, 5 and 6. Directions of currents are arbitrarily chosen.

(*) The incidence matrix is formed by taking nodes as rows and branches as columns

Nodes	Branches					
	1	2	3	4	5	6
A	-1	+1	+1	0	0	0
B	0	-1	0	-1	+1	0
C	0	0	-1	+1	0	+1
D	+1	0	0	0	-1	-1

$$P = \begin{pmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 & -1 \end{pmatrix}$$

In the above example the fourth row is negative of sum of the first three rows. Hence the fourth can be eliminated as we know that it can be obtained by negative sum of first three rows. As a result of this we get the reduced incidence matrix.

$$PR = \begin{pmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 \end{pmatrix}$$

Properties of a complete incidence matrix

(*) Sum of the entries of each column is zero
(*) Rank of the matrix is (n-1), where n is the no of nodes
(*) Determinant of a loop of complete incidence matrix is always zero

Example 2 : The incidence matrix of a graph is as shown. Draw the corresponding graph.

Solution :
The sum of each column of the given matrix is zero. Hence it is a compete incidence matrix.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & -1 \\ -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 & -1 & 0 \end{pmatrix}$$

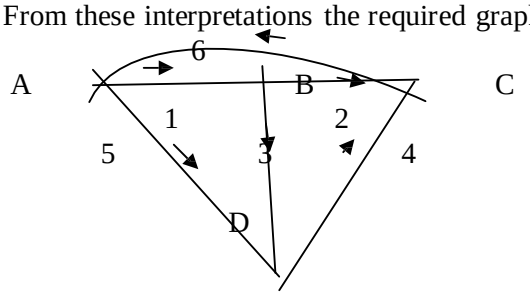
Number of nodes = n = 4 (say A. B. C and D)
Number of branches = b = 6 (say 1, 2, 3, 4, 5 and 6)

Prepare a tabular column as shown.

Nodes	Branches					
	1	2	3	4	5	6
A	1	0	0	0	1	-1
B	-1	1	1	0	0	0
C	0	-1	0	-1	0	1
D	0	0	-1	1	-1	0

From the tabular column, the entries have to be interpreted as follows:

From the first column the entries for A and B are one's . Hence branch 1 is connected between nodes A and B . Since for node A entry is +1 and for node B it is -1, the current leaves node A and enters node B and so on.
From these interpretations the required graph is drawn as shown.



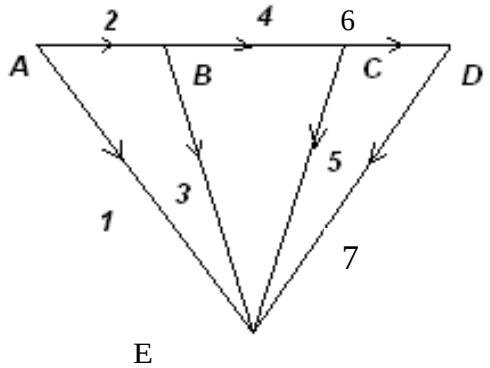
Example 3: The incidence matrix of a graph is as shown. Obtain the corresponding graph

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$

Solution:- Given incidence matrix is a reduced incidence matrix as the sum of each column is not zero. Hence it is first converted in to a complete incidence matrix by adding the deleted row. The elements of each column of the new row is filled using the fact that sum of each column of a complete incidence matrix is zero.
In the given matrix in first, third, fifth and the seventh column the sum is made zero by adding –1 in the new row and the corresponding node is E. The complete incidence matrix so obtained and also the graph for the matrix are as shown.

Nodes	Branches						
	1	2	3	4	5	6	7
A	1	1	0	0	0	0	0
B	0	-1	1	1	0	0	0
C	0	0	0	-1	1	1	0
D	0	0	0	0	0	-1	1
E	-1	0	-1	0	-1	0	-1

Graph:

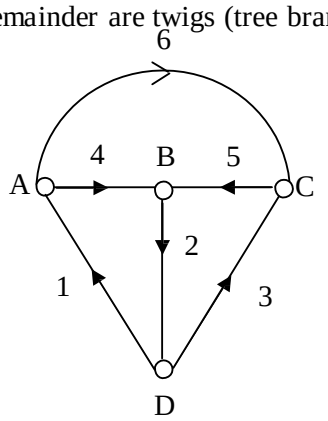


Tie – set Analysis:

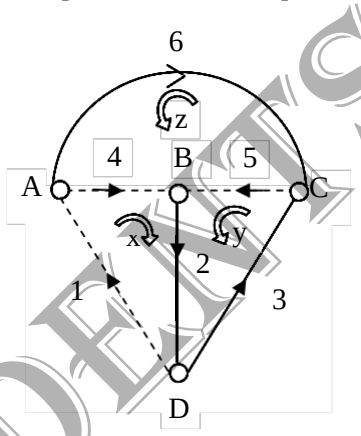
In order to form a tree from a network several branches need to be removed so that the closed loops open up. All such removed branches are called links and they form a Co-tree. Alternatively when a link is replaced in a tree, it forms a closed loop along with few of the tree branches. A current can flow around this closed loop. The direction of the loop current is assumed to be the same as that of the current in the link. The tree – branches and the link that form a loop is said to constitute a tie – set.

Definition

A tie – set is a set of branches contained in a loop such that the loop has at least one link and the remainder are twigs (tree branches)



Graph



tree (In thick lines) Co-tree (In dotted lines)

We see that by replacing the links 1, 4 and 5 three loops are formed and hence three loop currents x, y and z flow as shown. The relationships obtained between loop currents, tree branches and links can be scheduled as follows

Tie – set schedule

Tie – set	Tree – branches	Link	Loop current
1	2, 3	5	x
2	1, 4	2	y
3	4, 5	6	z

Tie – set matrix (Bf)

The Tie – set schedule shown above can be arranged in the form of a matrix where in the loop currents constitute the rows and branches of the network constitute the columns Entries inside the matrix are filled by the following procedure :

Let an element of the tie – set matrix be denoted by mik

Then mik = 1 Branch K is in loop i and their current directions are same
-1 If branch K is in loop i and their current directions are opposite.
If the branch K is not in loop i

By following this procedure we get the Tie – set matrix which is shown below:

Loop currentss	Branches					
	1	2	3	4	5	6
x	0	+1	+1	0	+1	0
y	+1	+1	0	+1	0	0
z	0	0	0	+1	-1	-1

Or

$$Bf = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 \end{pmatrix}$$

Analysis of the net work based on Tie – set schedule

From the tie –set schedule we make the following observations.

(i) Column wise addition for each column gives the relation between branch and loop currents

That is i1 = y
i2 = x + y
i3 = x
i4 = y+ z
i5 = x - z
i6 = - z

Putting the above equations in matrix form, we get

$$\begin{matrix} i1 \\ i2 \\ i3 \\ i5 \\ i6 \end{matrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 \\ z \end{pmatrix} \quad y$$

In compact form IB = B fT IL

Where IB = Branch current matrix
B fT = Transpose of the tie- set matrix
IL = Loop current column matrix
(ii)Row wise addition for each row gives the KVL equations for each fundamental loop

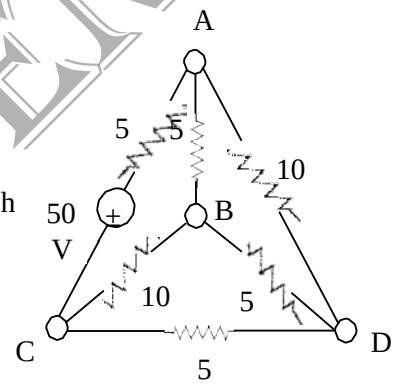
Row - 1 : V1 + V2 + V3 = 0
Row - 2 : V1 + V2 + V4 = 0
Row - 3 : V4 - V5 - V6 = 0

$$\begin{array}{c} \\ \\ \\ V5 \end{array} \begin{array}{c} \begin{array}{cccccc} & & V1 & & & \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 \end{pmatrix} \end{array} \begin{pmatrix} V2 \\ V3 \\ V4 \\ V6 \end{pmatrix} = 0$$

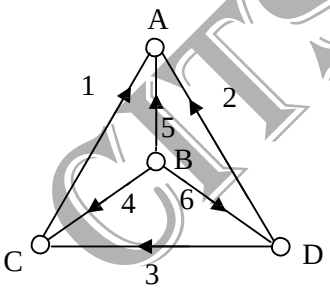
In compact form B f VB = 0 - - - (1)

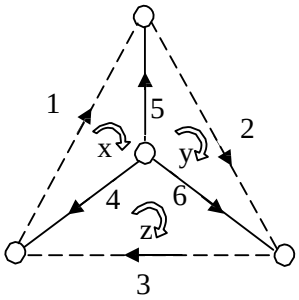
Where VB = Branch voltage column matrix and
B f = Tie - set matrix

Example: For the network shown in figure, write a Tie-set schedule and then find all the branch Currents and voltages



Solution: The graph and one possible tree is shown:





Loop current	Branch Numbers 1	2	3	4	5	6
x	1	0	0	1	-1	0
y	0	1	0	0	1	-1
z	0	0	1	-1	0	1

Tie set Matrix: Bf =

$$\begin{pmatrix} 1 & 0 & 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 1 & -1 & \\ 0 & 1 & -1 & 0 & 1 & \end{pmatrix}$$

Branch Impedence matrix ZB = $\begin{pmatrix} 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 \end{pmatrix}$

$$ZL = \begin{pmatrix} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

$$ZL = \begin{pmatrix} 20 & -5 & -10 \\ -5 & 20 & -5 \\ -10 & -5 & 20 \end{pmatrix}$$

Loop Equations : ZL IL = - Bf Vs

$$\begin{pmatrix} 20 & -5 & -10 \\ -5 & 20 & -5 \\ -10 & -5 & 20 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = - \begin{pmatrix} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} -50 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$20x-5y-10z = 50$$

$$-5x+20y-5z = 0$$

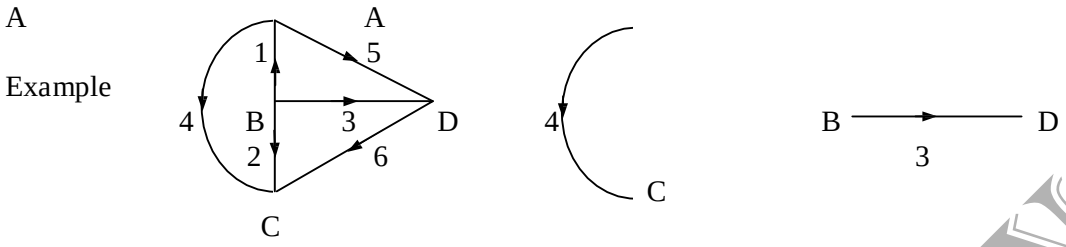
$$-10x-5y+20z = 0$$

Solving the equations, we get x = 4.17 Amps
y = 1.17 Amps
And z = 2.5 Amps

Cut – set Analysis

A cut – set of a graph is a set of branches whose removal , cuts the connected graph in to two parts such that the replacement of any one branch of the cut set renders the two parts connected.

Example



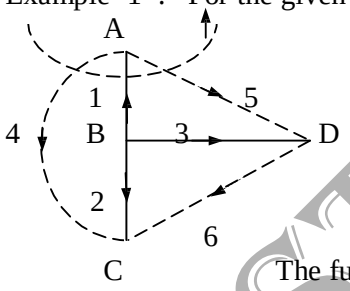
Directed graph Two separate graphs created by the cut set (1, 2, 5, 6)

Fundamental cut – set is a cut – set that contains only one tree branch and the others are links

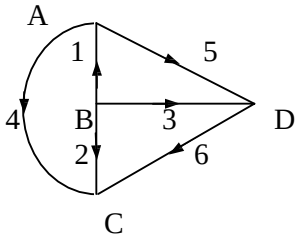
Formation of Fundamental cut – set

(*) Select a tree
(*) Select a tree branch
(*) Divide the graph in to two sets of nodes by drawing a dotted line through the selected tree branch and appropriate links while avoiding interruption with any other tree – branches.

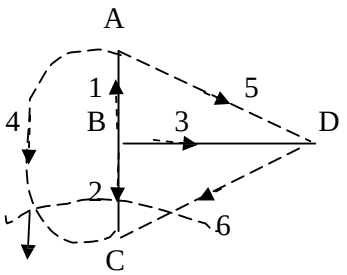
Example 1 : For the given graph write the cut set schedule



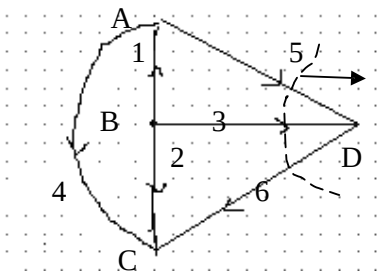
The fundamental cut –set of the Selected tree is shown in figure



Note that FCS - 1 yields node A and the set of nodes (B, C, D)
The Orientation of the fundamental cut – set is usually assumed to be the same as the orientation of the tree branch in it, Which is shown by an arrow. By following the same procedure the FCS- 2 and FCS -3 are formed as shown below:



FCS -2



FCS -3

It should be noted that for each tree branch there will be a fundamental cut – set. For a graph having 'n' number of nodes the number of twigs is (n-1).Therefore there will be (n-1) (n-1) fundamental cut-sets.

Once the fundamental cut sets are identified and their orientations are fixed, it is possible to write a schedule, known as cut – set schedule which gives the relation between tree – branch voltages and all other branch voltages of the graph.

Let the element of a cut – set schedule be denoted by Q_{ik} then,

$$Q_{ik} = \begin{cases} 1 & \text{If branch K is in cut – set I and the direction of the current in the branch K is same as cut – set direction .} \\ -1 & \text{If branch k is in cut – set I and the direction of the current in k is opposite to the cut – set direction.} \\ O & \text{If branch is k is not in cut – set i} \end{cases}$$

Cut set Schedule

Tree branch voltage	Branch Voltages					
	1	2	3	4	5	6
e_1	1	0	0	-1	-1	0
e_2	0	1	0	1	0	1
e_3	0	0	1	0	1	-1

The elements of the cut set schedule may be written in the form of a matrix known as the cut set matrix.

$$Q_f = \begin{pmatrix} 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{pmatrix}$$

<u>Analysis of a network using cut set schedule</u>

(*) Column wise addition of the cut set schedule gives the relation between tree branch voltage and the branch voltages for the above cut the schedule

$$V_1 = e_1$$
$$V_2 = e_2$$
$$V_3 = e_3$$
$$V_4 = -e_1 + e_2$$
$$V_5 = -e_1 + e_3$$
$$V_6 = e_2 - e_3$$

In matrix form

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}$$

In compact form

$V_B = Q^T{}_f V_T$ ……. (1) Where V_B =Branch voltage Matrix
$Q^T{}_f$ = Transpose of cut set matrix
and V_T = Tree branch voltage matrix.

(*) Row wise addition given KCL at each node

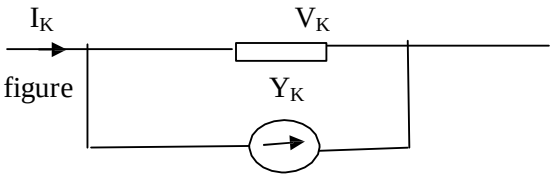
$$I_1 - I_4 - I_5 = 0$$
$$I_2 + I_4 + I_6 = 0$$
$$I_3 + I_5 + I_6 = 0$$

In matrix form

$$\begin{pmatrix} 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{pmatrix} = 0$$

In compact form $Q_f\ I_B = 0$ ……(2) Where Q_f = cut set matrix
I_B = Branch current matrix

(*) Let us consider a network having 'b' branches.
Each of the branches has a representation as shown in figure



I_{SK}

Referring to the figure $I_k = Y_k \; V_k + Is_k$

Since the network has b branches, one such equation could be written for every branch of the network ie $I_1 = Y_1 \; V_1 + Is_1$

$I_2 = Y_2 \; V_2 + Is_2$

.....

$I_b = Y_b \; V_b + Is_b$

Putting the above set of equations in a matrix form we get

$$I_B = Y_B \; V_B + Is \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (3)$$

I_B = Branch current matrix of order bx1

$$Y_B = \begin{pmatrix} Y_1 & 0 & 0 & - & - & 0 \\ 0 & Y_2 & 0 & - & - & 0 \\ - & - & - & - & - & - \\ 0 & 0 & - & - & - & Y_b \end{pmatrix} = \text{Branch admittance matrix of order bx b}$$

V_B = Branch Voltage column matrix of order bx1 and
I_B = Source current matrix of order bx1.

Substituting (3) in (2) we get $Q_f \left[Y_B \; V_B + I_S \right] = 0$

$$Q_f \; Y_B \; V_B + Q_f \; I_S = 0$$

But from (1) we have $V_B = Q_f^{\,T} \; V_T$

Hence equation (4) becomes

$$Q_f \; Y_B \; Q_f^{\,T} \; V_T + Q_f \; Is = 0$$
$$Y_C \; V_T + Q_f Is = 0$$

or $Y_C \; V_T = - Q_f \; I_S$ ..... (5)

Where $Y_C = Q_f \; Y_B \; Q_f^{\,T}$ is called cut - set admittance matrix

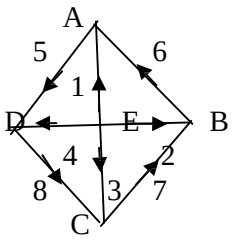
<u>Action Plan for cut – set analysis.</u>

(*) Form the cut- set matrix Q_f
(*) Construct the branch admittance matrix Y_B
(*) Obtain the cut – set admittance matrix using the equation $Y_C = Q_f \; Y_B \; Q_f^{\,T}$
(*) Form the KCL or equilibrium equations using the relation $Y_C \; V_T = - Q_f \; I_S$

The elements of the source current matrix are positive if the directions of the branch current and the source connect attached to that branch are same otherwise negative.

(*) The branch voltages are found using the matrix equation $V_B = Q_f^T \; V_T$
(*) Finally the branch currents are found using the matrix equation $I_B = Y_B \; V_B + I_S$

Example 2 : For the directed graph obtain the cut set matrix



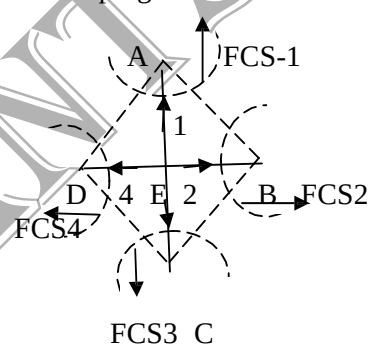
Solution : The tree (marked by thick lines)and the link (marked by doffed lines)are as shown. The fundamental cut sets are formed at nodes A B C and D keeping 'E' as reference node

Fcs - 1 -- (1, 5, 6)

Fcs - 2 -- (2, 6, 7)

Fcs - 3 -- (3, 7, 8)

Fcs - 4 -- (4, 5, 8)

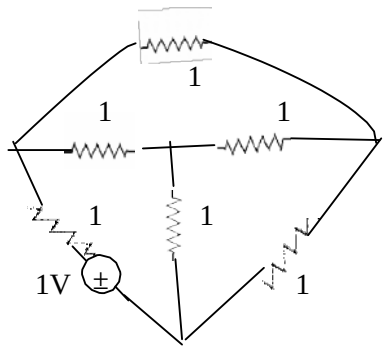


Hence the cut – set schedule is as follows:

Tree branch voltage	Branch							
	1	2	3	4	5	6	7	8
e_1	1	0	0	0	-1	1	0	0
e_2	0	1	0	0	0	-1	1	0
e_3	0	0	1	0	0	0	-1	1
e_4	0	0	0	1	1	0	0	-1

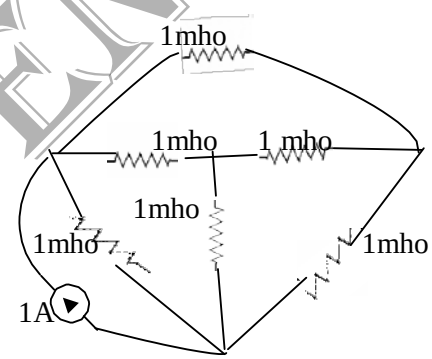
Hence the required cut –set matrix $Q_f = \begin{pmatrix} 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & -1 \end{pmatrix}$

Example 3: Find the branch voltages using the concept of cut-sets



<u>Solution :</u> The voltage source is Transformed in to an equivalent current source. It should be noted that all the circuit Passive elements must be admittances and the net work should contain only current sources.

The graph for the network is shown. A possible tree (shown with thick lines) and co tree (shown by dotted lines) are shown



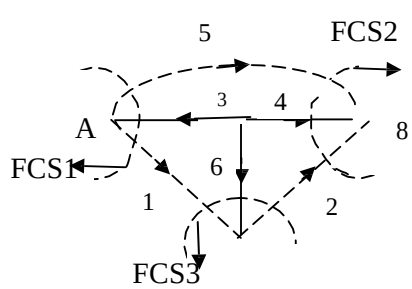
FCS 1 = (3, 1, 5)

FCS 2 = (4, 2, 5)

FCS 3 = (6, 1, 2)

Cut set schedule

Tree branch voltage	Branches					
	1	2	3	4	5	6
e_3	-1	0	1	0	-1	0
e_4	0	1	0	1	1	0
e_6	1	-1	0	0	0	1



$$Q_f = \begin{pmatrix} -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$Y_B = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$Y_C = \begin{pmatrix} -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{pmatrix}$$

<u>Equilibrium Equations</u> ; $Y_C \; V_T = - Q_f \; I_S$

$$\begin{pmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{pmatrix} \begin{pmatrix} e_3 \\ e_4 \\ e_6 \end{pmatrix} = - \begin{pmatrix} -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$3\,e_3 - e_4 - e_6 = -1$

$-e_3 + 3\,e_4 - e_6 = 0$

$-\,e_3 - e_4 + 3e_6 = 1$

Solving we get e_3=-0.25 volt

e_4=0

e_6=0.25 volt

DUALITY CONCEPT

Two electrical networks are duals if the mesh equations that characterize one have the same mathematical form as the nodal equations of the other.

Example 1

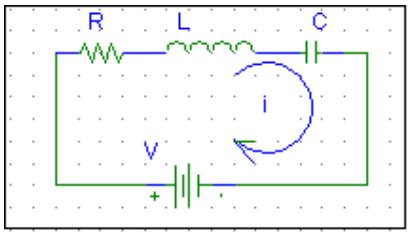


Figure 1

Consider an R-L-C series network excited by a voltage so source V as shown in the figure. The equation generating
the circuit behavior is $Ri+L\frac{di}{dt}+\frac{1}{C}\int idt=V$ ……..(1)

Now consider the parallel G-C-L network fed by a
Current Source i is shown in the figure. The equation generating the
Circuit behavior is $GV+C\frac{dV}{dt}+\frac{1}{L}\int vdt=i$ ……..(2)

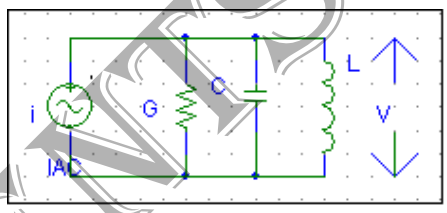


Figure 2

Comparing the equations (1) and (2),we get the similarity between the networks of fig(1) and fig(2).The solution of equation (1) will be identical to the solution of equation (2) when the following exchanges are made
R --- G, L--- C, C---L and V ---i
Hence networks of figure (1) and (2) are dual to each other.

Table of dual Quantities

1.Voltage Source	Current source
2.Loop currents	Node voltages
3.Iductances	Capacitances
4.Resistances	Conductances
5.Capacitances	Inductances
6. On KVL basis	On KCL basis
7.Close of switch	Opening of switch

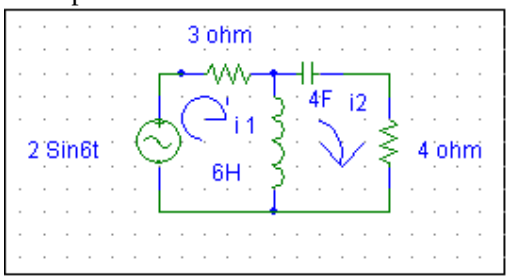
Note:Only planar networks have duals.

Procedure for drawing dual network
The duals of planar networks could be obtained by a graphical technique known as the dot method. The dot method has the following procedure.

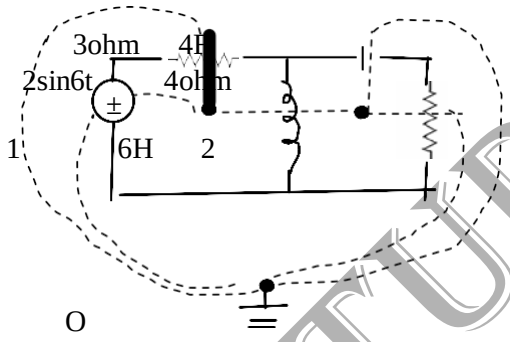
Put a dot in each independent loop of the network. These dots correspond to independent nodes in the dual network.

Put a dot outside the network. This dot corresponds to the reference node in the dual network.
Connect all internal dots in the neighboring loops by dotted lines cutting the common branches. These branches that are cut by dashed lines will form the branches connecting the corresponding independent nodes in the dual network.
Join all internal dots to the external dot by dashed lines cutting all external branches. Duals of these branches will form the branches connecting the independent nodes and the reference node.

Example 1:



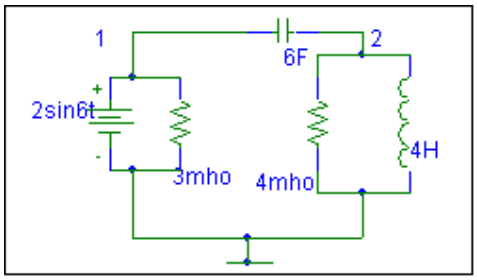
Draw the exact dual of the electrical circuit shown in the figure.



Solution: Mark two independent nodes 1 and 2 and a reference node 0as shown in the figure.
Join node 1 and 2 by a dotted line passing through the inductance of 6H.this element will appear as capacitor of 6 F between node 1 and 2 in the dual.
Join node 1 and reference node through a dotted line passing the voltage source of 2sin6t volts. This will appear as a current source of 2sin6t amperes between node1 and reference node.

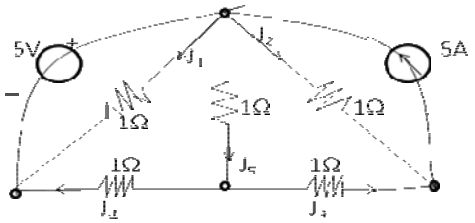
Join node 1 and reference node through a dotted line passing through 3 ohms resistor. This element appears as 3mho conductance between node1 and reference node in the dual.
Join node 2 and reference node through a dotted line passing through the capacitor of 4 Farads. This element will appear as 4 Henry inductor between node 2 and reference node in the dual
Join node 2 and reference node through a dotted line passing through the resistor of 4 ohms. This element will appear as 4 mho conductance between node 2 and reference node.

The Dual network drawn using these procedural steps is shown.

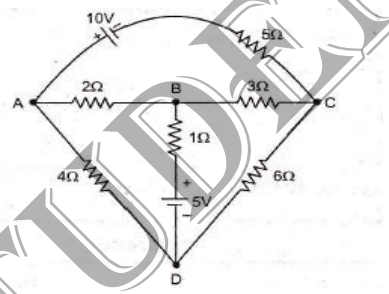


Assignment questions:

1) Explain incidence matrix of a network of a network graph ? Give suitable example.
2) Define the following with suitable examples
   i) Planar and non-planar graph
   ii) Twigs and links
3) For the network shown in Fig. 8 write the graph of the network and obtain the tie-set schedule considering J_1, J_2, J_5 as tree branches. Calculate all branch current.



4) Explain briefly trees, cotrees, and loops in a graph of network with suitable example
5) Explain with examples the principal of duality
6) Draw the oriented graph of the network shown in Fig. 10 select a tree, write the set schedule and obtain equilibrium equations



7) under what conditions do you consider topology for network analysis? For the graph shown in Fig. 3, for a co- tree (4,5,2,8), write tie set and cut set matrices. (10)

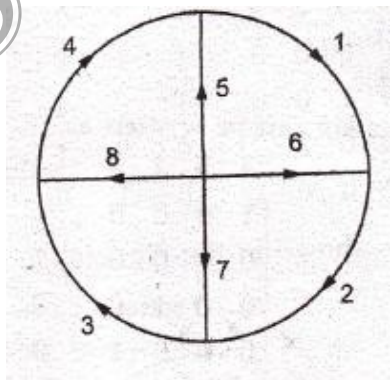


Fig. 3

8) For the network shown in Fig. 4, draw its dual. Write in intergo differential form
i) mesh equations for the given network ii) node equations for the dual.

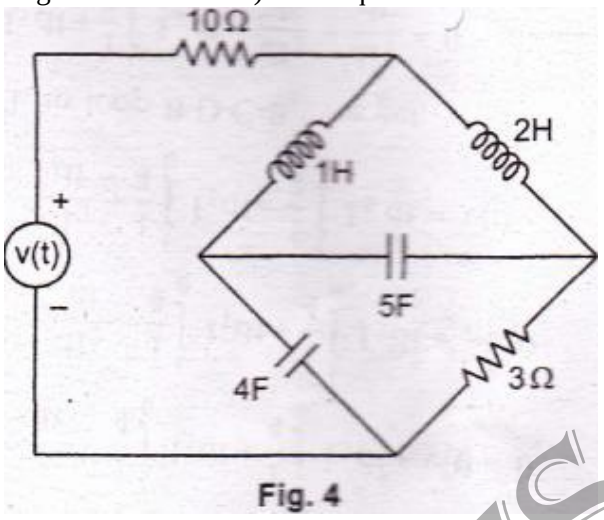


Fig. 4

V(t) = 10 sin 40t.

9) What are dual networks ? what is their significance ? Draw the dual of the circuit shown in fig. 8 .

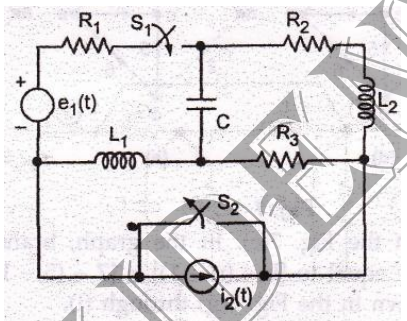


Fig. 8

10) For the network shown in Fig. 5, perform source shifts, draw a graph, select tree with branches 1,2 and 3 and obtain tie set and matrices.

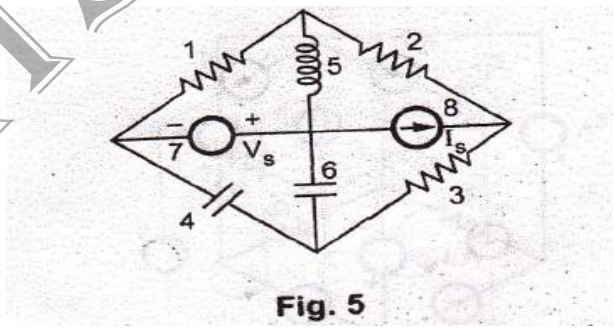


Fig. 5

Unit: 3 Network Theorems – 1: Hrs: 06

Syllabus of unit :
Superposition, Reciprocit y and Millman's theorems

Recommended readings:

1. "Network Analysis", M. E. Van Valkenburg, PHI / Pearson Education

2. "Networks and systems", Roy Choudhury, 2 edition, New Age International Publications .

3. "Network theory ", Ganesh Rao.

4. "Network analysis" , Roy Choudry.

NETWORK THEOREMS

Mesh current or node voltage methods are general methods which are applicable to any network. A number of simultaneous equations are to be set up. Solving these equations, the response in all the branches of the network may be attained. But in many cases, we require the response in one branch or in a small part of the network. In such cases, we can use network theorems, which are the aides to simplify the analysis. To reduce the amount of work involved by considerable amount, as compared to mesh or nodal analysis. Let us discuss some of them.

SUPERPOSITION THEOREM:

The response of a linear network with a number of excitations applied simultaneously is equal to the sum of the responses of the network when each excitation is applied individually replacing all other excitations by their internal impedances.

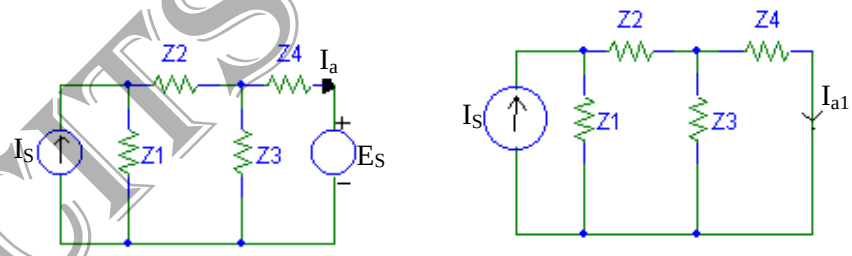
Here the excitation means an independent source. Initial voltage across a capacitor and the initial current in an inductor are also treated as independent sources.

This theorem is applicable only to linear responses and therefore power is not subject to superposition.

During replacing of sources, dependent sources are not to be replaced. Replacing an ideal voltage source is by short circuit and replacing an ideal current source is by open circuit.

"In any linear network containing a number of sources, the response (current in or voltage across an element) may be calculated by superposing all the individual responses caused by each independent source acting alone, with all other independent voltage sources replaced by short circuits and all other independent current sources replaced by open circuits". Initial capacitor voltages and initial inductor currents, if any, are to be treated as independent sources.

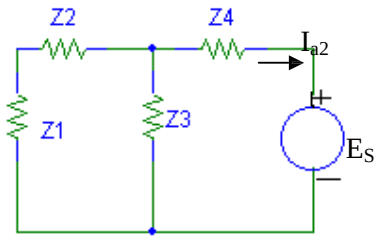
To prove this theorem consider the network shown in fig.



We consider only one-voltage sources and only one current sources for simplicity. It is required to calculate Ia with Is acting alone the circuit becomes

$$Ia_1 = \left[\frac{I_S \quad Z_1}{Z_1 + Z_2 + \dfrac{Z_3\, Z_4}{Z_3 + Z_4}}\right]\left[\frac{Z_3}{Z_3 + Z_4}\right]$$

$$= I_S \frac{Z_1\, Z_3}{(Z_1 + Z_2 + Z_3)\, Z_4 + (Z_1 + Z_2)\, Z_3} \qquad \text{----------(1)}$$

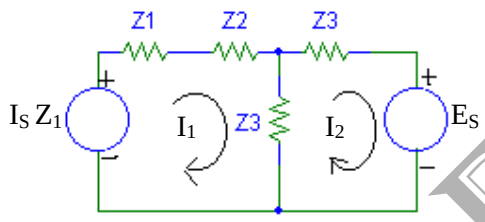


with E_S acting alone

$$Ia_1 = \frac{-E_S}{\frac{Z_4 + (Z_1 + Z_2) Z_3}{Z_1 + Z_2 + Z_3}}$$

$$= \frac{-E_S (Z_1 + Z_2 + Z_3)}{(Z_1 + Z_2 + Z_3) Z_4 + (Z_1 + Z_2) Z_3} \quad \text{----------(2)}$$

Next converting the current source to voltage source, the loop equations



$$I_2 = \frac{\begin{vmatrix} Z_1+Z_2+Z_3 & I_S Z_1 \\ -Z_3 & -E_S \end{vmatrix}}{\begin{vmatrix} Z_1+Z_2+Z_3 & -Z_3 \\ -Z_3 & Z_3+Z_4 \end{vmatrix}}$$

$$= \frac{I_S Z_1 Z_3 - E_S (Z_1+Z_2+Z_3)}{(Z_1+Z_2+Z_3) Z_4 + (Z_1+Z_2) Z_3} \quad \text{----------(3)}$$

From equation (1), (2) and (3) $Ia_1 + Ia_2 = I_2 = Ia$

Hence proof

Reciprocity Theorem :

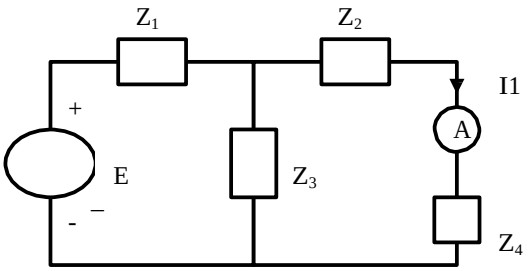
In an initially relaxed linear network containing one independent source only. The ratio of the response to the excitation is invariant to an interchange of the position of the excitation and the response.

i.e if a single voltage source Ex in branch X produces a current response I_y the branch Y, then the removal of the voltage source from branch x and its insertion in branch Y will produce the current response I_y in branch X.

Similarly if the single current source Ix between nodes X and X′ produces the voltage response Vy between nodes Y and Y′ then the removal of the current source from X and X′ and its insertion between Y and Y′ will produce the voltage response Vy between the nodes X and X′.

Between the excitation and the response, one is voltage and other is current. It should be noted that after the source and response are interchanged, the current and the voltages in other parts of the network will not remain the same.

Proof :



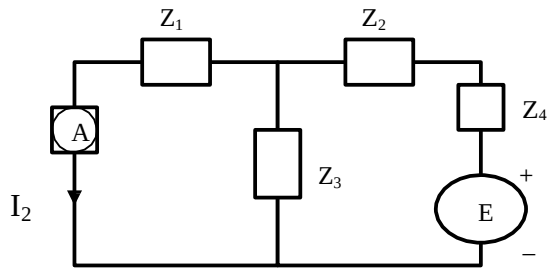
Consider a network as shown in which the excitation is E and the response is I in Z4. The reading of the ammeter is

$$I_1 = \frac{E}{Z_1 + \dfrac{Z_3\,(Z_2 + Z_4)}{Z_2+Z_3+Z_4}} \cdot \frac{Z_3}{Z_2+Z_3+Z_4}$$

$$I_1 = \frac{E\,Z_3}{Z_1\,(Z_2+Z_3+Z_4) + Z_3(Z_2 + Z_4)} \quad \text{..... (1)}$$

Next interchange the source and ammeter.

E



Now the reading of the Ammeter is :

$$I_2 = \frac{E}{(Z_2 + Z_4) + \frac{Z_1 Z_3}{Z_1 + Z_3}} \cdot \frac{Z_3}{Z_1+Z_3}$$

$$I_2 = \frac{E Z_3}{Z_1 (Z_2+Z_3+Z_4) + Z_3(Z_2 + Z_4)} \quad \text{..... (2)}$$

From (1) & (2)

$$I_1 = I_2$$

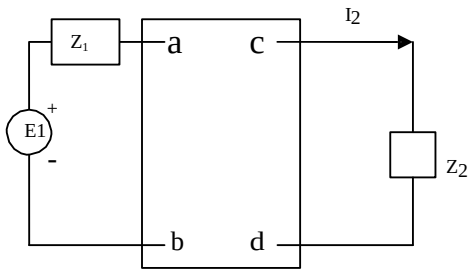
It can be similarly be shown for a network with current sources by writing node equations.

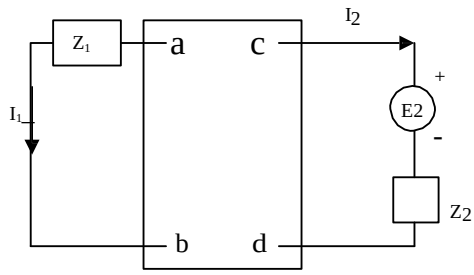
Transfer Impedance :

The transfer impedance between any two pairs of terminals of a linear passive network is the ratio of the voltage applied at one pair of terminals to the resulting current at the other pair of terminals .

With this definition the reciprocity theorem can be stated as :

"Only one value of transfer impedance is associated with two pairs of terminals of a linear passive network " .





w.r.t figs shown $\frac{E1}{I_2} = \frac{E2}{I_1} = ZT$

If E1 = E2 then I1 = I2.

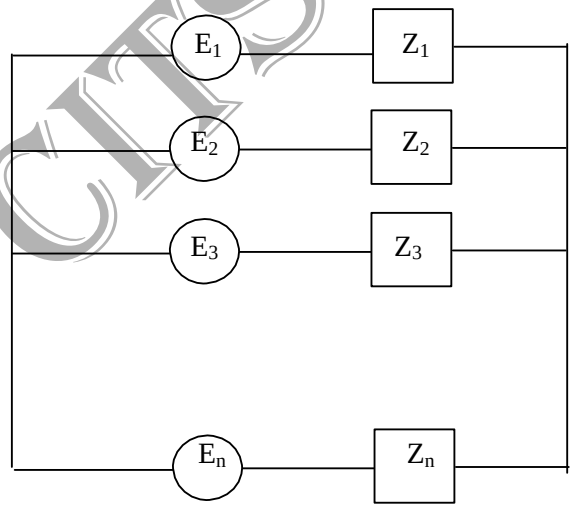
Millman's Theorem:

Certain simple combinations of potential and current source equivalents are of use because they offer simplification in solutions of more extensive networks in which combinations occur. Millman's Theorem says that "if a number of voltage sources with internal impedances are connected in parallel across two terminals, then the entire combination can be replaced by a single voltage source in series with single impedance".

The single voltage is the ratio

$$\frac{\text{Sum of the product of individual voltage sources and their series admittances}}{\text{Sum of all series admittances}}$$

and the single series impedance is the reciprocal of sum of all series admittances.



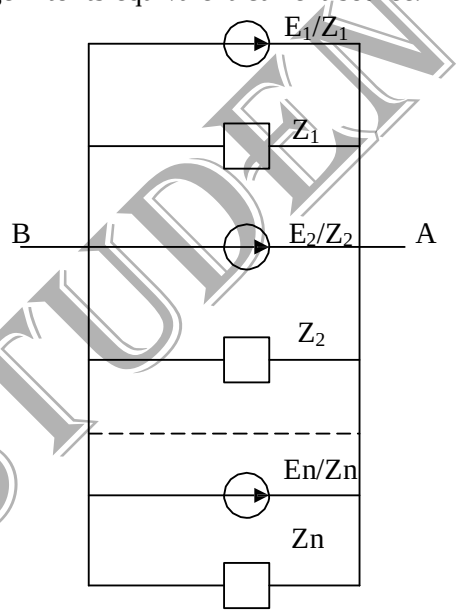
Let E_1, E2……E_n be the voltage sources and Z_1, Z2……Z_n are their respective impedances. All these are connected between A & B with Y=1/Z, according to Millman's Theorem, the single voltage source that replaces all these between A & B is

$$E_{AB} = \frac{\sum_{K=1}^{n} E_K Y_K}{\sum_{K=1}^{n} Y_K}$$

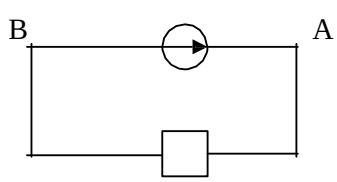
And

The single impedance is $Z = \dfrac{1}{\sum_{K=1}^{n} Y_K}$

Proof: Transform each voltage into its equivalent current source. Then the circuit is as in Fig.



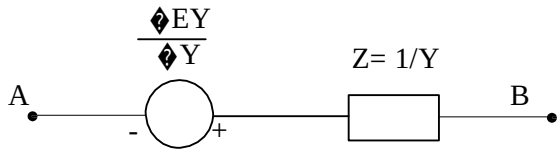
With Y=1/Z the circuit is simplified as E_1 Y1+E_2 Y2 +……..EnYn= $\sum E_K Y_K$



Y1+ Y2 +……..Yn= $\sum Y_K$

Which is a single current source in series with a single admittance

Retransforming this into the equivalent voltage source



The theorem can be stated as "If a number of current sources with their parallel admittances are connected in series between terminals A and B, then they can be replaced by a single current source in parallel with a single admittance. The single current source is the ratio

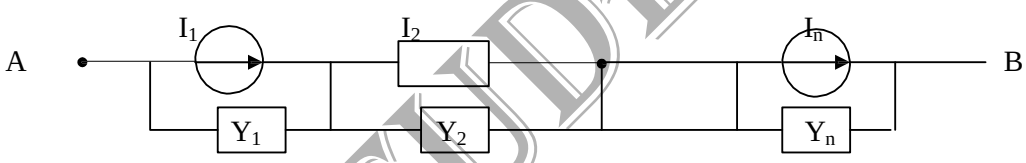
$$\frac{\text{Sum of products of individual current sources and their impedances}}{\text{Sum of all shunt impedances}}$$

And the single shunt admittance is the reciprocal of the sum of all shunt impedances.

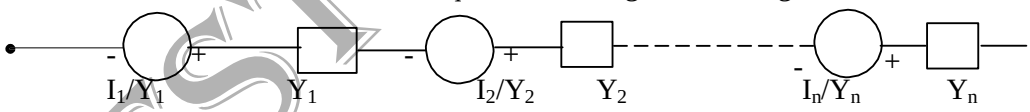
Let I_1, I_2, ……..I_n be the n number of current sources and Y_1,Y_2……..Yn be their respective shunt admittances connected in series between A & B. Then according to Millman's Theorem they can be replaced by single current I_{AB} in parallel with a single admittance Y_{AB} where $I_{AB}=$ $I_K Z_K$

Z_K

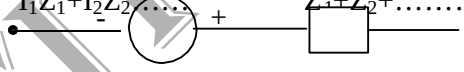
And $Y_{AB}=\dfrac{1}{Z_K}$



Transform each current source into its equivalent voltage source to get the circuit as in fig A B

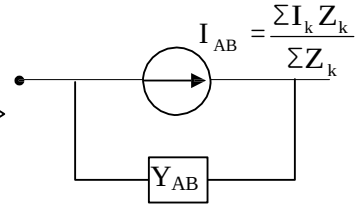


= $I_1Z_1+I_2Z_2$…… Z_1+Z_2+……..



$$I_{AB}=\frac{\Sigma I_k Z_k}{\Sigma Z_k}$$

Retransforming to equivalent current source

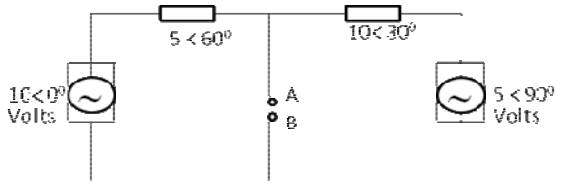


$$=\frac{1}{\Sigma Z_k}$$

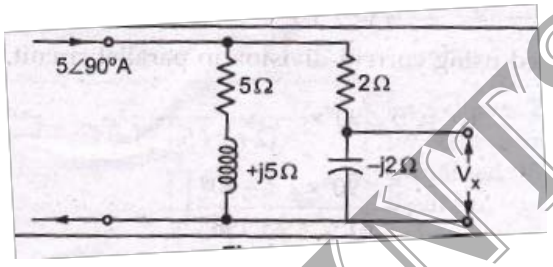
Assignment questions:

1) Find the condition for maximum power transfer in the following network type AC source , complex source impedance and complex load impedance but only load resistance varying.

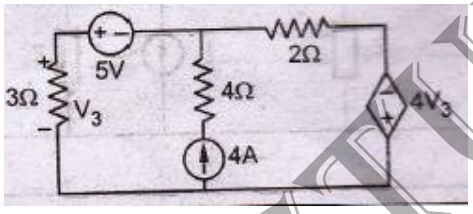
2)In the circuit shown in fig .10 find the load connected at AB for which the power transferred will be maximum . Also find maximum power



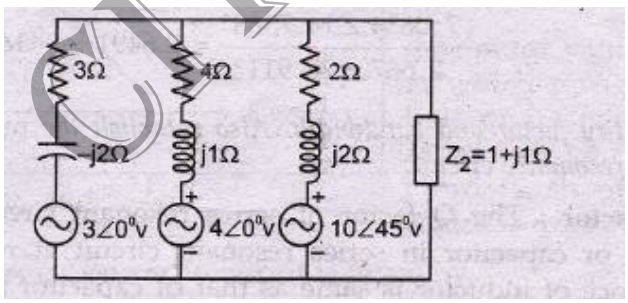
3) In the circuit shown in Fig. 11. Find V_x and prove reciprocity theorem.



4)) Determine the current through 2Ω resister of the network shown in Fig. using superposition Principle



5) State Millman's theorem, using the same calculate current through the load in the circuit shown in Fig.



7) Calculate the current I shown in fig. 19 using Millman's theorem

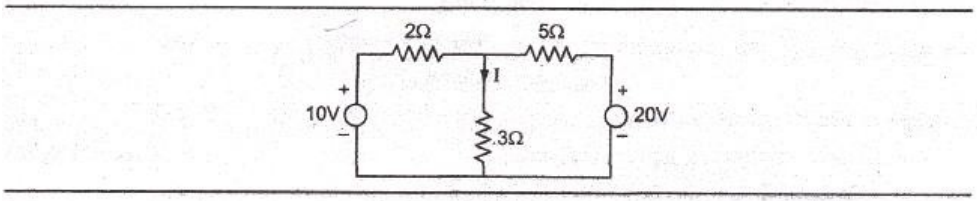


Fig. 19

8) state and explain i) Reciprocity theorem ii) Millmann's theorem.

Using superposition theorem, obtain the response I for the network shown in Fig.12.

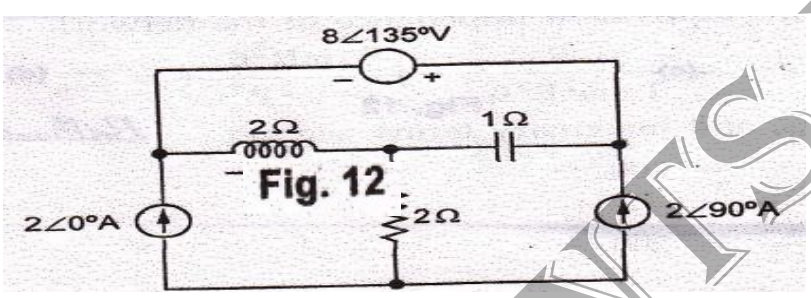


Fig. 12

9) Use millman's theorem to determine the voltage Vs of the network shown in Fig.6 Given that

$E_R = 230 \angle 0^0$ V, $E_Y = 230 \angle -120^0$V, and $E_B = 230 \angle 120^0$V

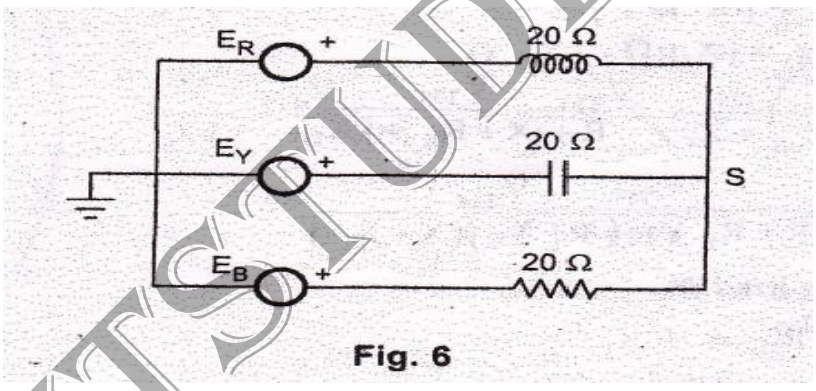


Fig. 6

Unit: 4 Network Theorems - II : Hrs: 06

Syllabus of unit :

Thevinin's and Norton's theorems; Maximum Power transfer theorem

Recommended readings:

1. "Network Analysis", M. E. Van Valkenburg, PHI / Pearson Education

2. "Networks and systems", Roy Choudhury, 2 edition, New Age International Publications .

3. "Network theory ", Ganesh Rao.

4. "Network analysis" , Roy Choudry.

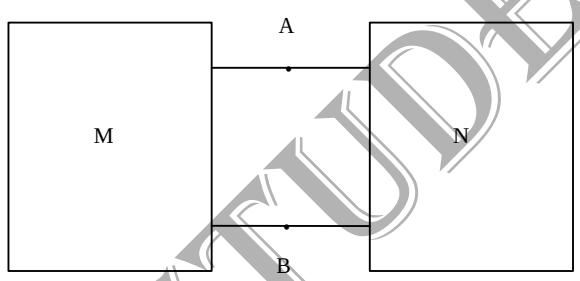
Thevinin's and Norton's Theorems:

If we are interested in the solution of the current or voltage of a small part of the network, it is convenient from the computational point of view to simplify the network, except that small part in question, by a simple equivalent. This is achieved by Thevinin's Theorem or Norton's theorem.

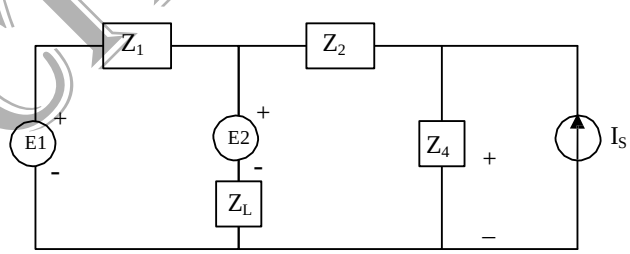
Thevinin's Theorem :

If two linear networks one M with passive elements and sources and the other N with passive elements only and there is no magnetic coupling between M and N, are connected together at terminals A and B, then with respect to terminals A and B, the network M can be replaced by an equivalent network comprising a single voltage source in series with a single impedance. The single voltage source is the open circuit voltage across the terminals A and B and single series impedance is the impedance of the network M as viewed from A and B with independent voltage sources short circuited and independent current sources open circuited. Dependent sources if any are to be retained.

Arrange the networks M and N such that N is the part of the network where response is required.



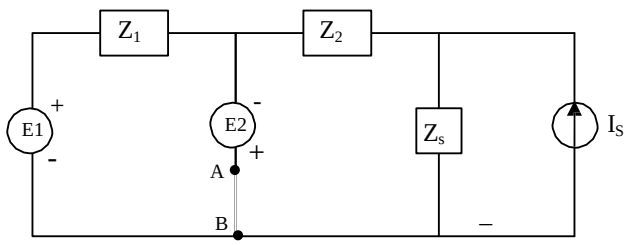
To prove this theorem, consider the circuit shown in Fig.



Suppose the required response is the current IL in ZL. Connected between A and B. According to Thevinin's theorem the following steps are involved to calculate IL

Step 1:

Remove ZL and measure the open circuit voltage across AB. This is also called as Thevinin's voltage and is denoted as VTH

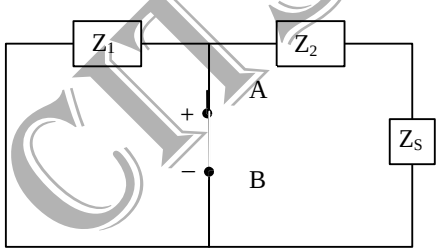


$$V_{TH} = V_{AB} = E_1 - \frac{E_1 - I_S Z_S}{Z_1 + Z_2 + Z_S} Z_1 + E_2$$

$$V_{TH} = V_{AB} = \frac{(E_1 + E_2)(Z_1 + Z_2 + Z_S) - (E_1 - I_S Z_S) Z_1}{Z_1 + Z_2 + Z_S}$$

Step 2:

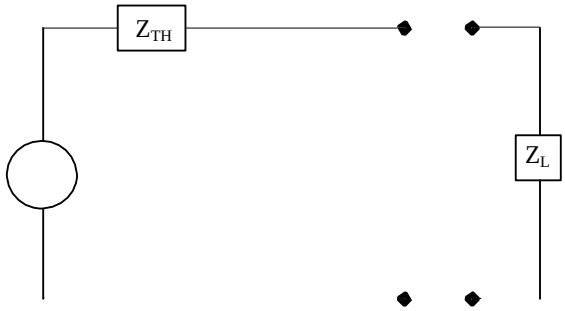
To obtain the single impedance as viewed from A and B, replace the network in Fig. replacing the sources. This single impedance is called Thevinin's Impedance and is denoted by Z_{TH}



$$Z_{TH} = \frac{Z_1 (Z_2 + Z_S)}{Z_1 + Z_2 + Z_S}$$

Step 3 :

Write the thevinin's network and re introduce ZL



Then the current in Z_L is

$$I_L = \frac{V_{TH}}{Z_{TH} + Z_L}$$

$$= \frac{\dfrac{(E_1 + E_2)(Z_1+Z_2+Z_S) - (E_1 - I_S Z_S) Z_1}{Z_1+Z_2+Z_S}}{\dfrac{Z_1(Z_2+Z_S)}{Z_1+Z_2+Z_S} + Z_L}$$

$$= \frac{(E_1 + E_2)(Z_1+Z_2+Z_S) - (E_1 - I_S Z_S) Z_1}{Z_1(Z_2+Z_S) + Z_2(Z_1+Z_2+Z_S)}$$

To verify the correctness of this, write loop equations for the network to find the current in ZL

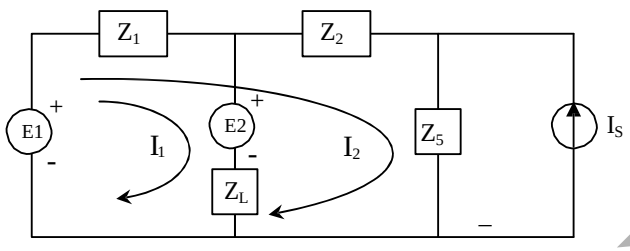
$$\begin{vmatrix} (E_1 + E_2) & Z_1 \\ (E_1 - I_S Z_s) & Z_1+Z_2+Z_S \end{vmatrix}$$

$$\begin{vmatrix} Z_1+Z_L & Z_1 \\ Z_1 & Z_1+Z_2+Z_S \end{vmatrix}$$

$$= \frac{(E_1 + E_2)(Z_1+Z_2+Z_S) - (E_1 - I_S Z_S) Z_1}{(Z_1 + Z_L)(Z_1+Z_2+Z_S) - Z_1^2}$$

$$= \frac{(E_1 + E_2)(Z_1+Z_2+Z_S) - (E_1 - I_S Z_S) Z_1}{Z_1(Z_2 + Z_S) + Z2\,(Z_1+Z_2+Z_S)}$$

Norton's Theorem :-



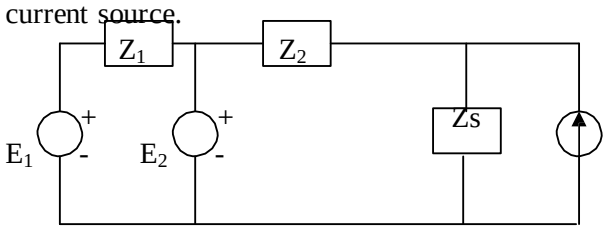
The Thevinins equivalent consists of a voltage source and a series impedance . If the circuit is transformed to its equivalent current source, we get Nortons equivalent. Thus Norton's theorem is the dual of the Thevinin's theorem.

If two linear networks, one M with passive elements and sources and the other N with passive elements only and with no magnetic coupling between M and N, are connected together at terminals A and B, Then with respect to terminals A and B, the network M can be replaced by a single current source in parallel with a single impedance. The single current source is the short circuit current in AB and the single impedance is the impedance of the network M as viewed from A and B with independent sources being replaced by their internal impedances

The proof of the Norton's theorem is simple

Consider the same network that is considered for the Thevinin's Theorem and for the same response.

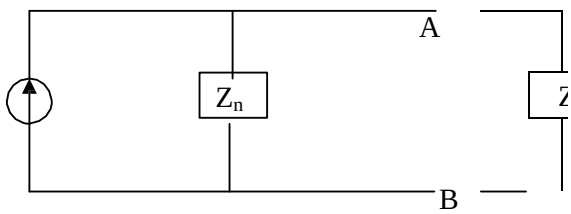
Step 1: Short the terminals A and B and measure the short circuit current in AB, this is Norton's current source.



$I_N = I_{sc} = \frac{E_1 + E_2}{} + \frac{E_2 + I_S Z_S}{}$

$$=\frac{(E_1+E_2)(Z_2+Z_S)+(E_2+I_S Z_S)Z_1}{Z_1(Z_2+Z_S)}\,Z_1 \qquad Z_2+Z_S$$

Step 2: This is the same as in the case of thevnin's theorem
Step 3: write the Nortons equivalent and reintroduce Z_L



Then the current in Z_L is

$$I_L = I_N.\frac{Z_n}{Z_n+Z_L}$$

$$= \frac{\dfrac{(E_1+E_2)(Z_2+Zs)+(E_2+I_sZ_s)Z_1}{Z_1(Z_2+Z_s)}\;.\;\dfrac{Z_1(Z_2+Z_s)}{Z_1+Z_2+Z_s}}{\dfrac{Z_1(Z_2+Z_s)}{Z_1+Z_2+Z_s}+Z_L}$$

$$= \frac{(E_1+E_2)(Z_2+Zs)+(E_2+I_sZ_s)Z_1}{Z_1(Z_2+Z_s)+Z_L(Z_1+Z_2+Z_s)}$$

$$= \frac{(E_1+E_2)(Z_1+Z_2+Zs)-(E_1-I_sZ_s)Z_1}{Z_1(Z_2+Z_s)+Z_L(Z_1+Z_2+Z_s)}$$

Verification is to be done as in Thevinin's Theorem

Determination of Thevinin's or Norton's equivalent when dependent sources are present

Since

$$I_L = \frac{V_{TH}}{Z_{TH}+Z_L} = \frac{I_N.Z_{TH}}{Z_{TH}+Z_L}$$

Z_{TH} can also be determined as $Z_{TH} = \dfrac{V_{TH}}{I_N} = \dfrac{\text{o.c voltage across AB}}{\text{s.c current in AB}}$

When network contains both dependent and independent sources. It is convenient to determine Z_{TH} by finding both the open circuit voltage and short circuit current

If the network contains only dependent sources both V_{TH} and I_N are zero in the absence of independent sources. Then apply a constant voltage source (or resultant source) and the ratio of voltage to current gives the Z_{TH}. However there cannot be an independent source ie, V_{TH} or I_N in the equivalent network.

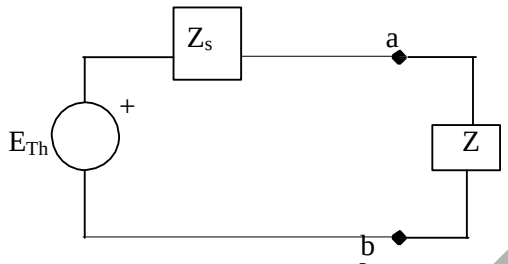
Maximum Transfer Theorem:-

When a linear network containing sources and passive elements is connected at terminals A and B to a passive linear network, maximum power is transferred to the passive network when its impedance becomes the complex conjugate of the Thevinin's impedance of the source containing network as viewed form the terminals A and B.

Fig represents a network with sources replaced by its Thevinin's equivalent of source of E_{TH} volts and impedance Z_s, connected to a passive network of impedance z at terminals A & B. With Z_s =Rs+JXs and z=R+JX, The proof of the theorem is as follows

Current in the circuit is

$$I = \frac{E_{TH}}{\sqrt{(Rs+R)^2+(Xs+X)^2}} \qquad (1)$$



power delivered to the load is $P=I^2R$

$$= \frac{E^2_{Th}}{(Rs+R)^2+(Xs+X)^2}.R \qquad (2)$$

As P = (R,X) and since P is maximum when dP=0

We have $dP= \frac{oP}{oR}.dR + \frac{oP}{oX}.dX$ (3)

power is maximum when $\frac{oP}{oR}=0$ and $\frac{oP}{oX}=0$ simultaneously

$$\frac{oP}{oR} = \frac{(Rs+R)^2+(Xs+X)^2 - R\{2(Rs+R)\}}{D^2} = 0$$

ie, $(Rs+R)^2+(Xs+X)^2 - 2R\{2(Rs+R)\} = 0$ (4)

$$\frac{oP}{oX} = \frac{-R\{2(Xs+X)\}}{D^2} = 0$$

ie $2R(Xs+X)=0$ (5)

From (5) we have X= -Xs (6)

Substituting in (4) $(Rs+R)^2 = 2R(Rs+R)$, ie, Rs+R= 2R

ie , R=Rs

Alternatively as $P = \dfrac{E^2R}{(Rs+R)^2 + (Xs+X)^2}$

$$= \frac{E^2 Z \cos\theta}{(Rs+Z\cos\theta)^2+(Xs+Z\sin\theta)^2}$$

$$= \frac{E^2 Z \cos\theta}{Zs^2+Z^2+2ZZs\cos(\theta-\theta s)} \qquad (7)$$

ie $P=f(Z,\theta)$

$$dP = \frac{\partial P}{\partial Z}.dZ + \frac{\partial P}{\partial \theta}.d\theta = 0$$

for Pmax

$$\frac{\partial P}{\partial Z} = 0 = \{Z_s^2+Z^2+2 Z Z_s \cos(\theta-\theta s)\} \cos\theta - Z \cos\theta \{2Z+2Z_s \cos(\theta-\theta s)\}$$

ie $Z_s^2+Z^2=2 Z^2+2Z Z_s \cos(\theta-\theta s)$. Or $|Z|=|Z_s|$ (8)

then with

$$\frac{\partial P}{\partial \theta} = 0 = \{Z_s^2+Z^2+2 Z Z_s \cos(\theta-\theta s)\}Z(-\sin\theta)-Z\cos\theta \{ZS^2+Z^2\ 2Z Z_s \sin(\theta-\theta_s)\}$$

$$(Z_s^2+Z^2)\sin\theta = 2Z Z_s \{\cos\theta \sin(\theta-\theta_s) - \sin\theta \cos(\theta-\theta_s)\}$$

$$= -2Z Z_s \sin\theta_s \qquad (9)$$

Substituting (8) in (9)

$$2 Z_s \sin\theta = -2 Z_s^2 \sin\theta_s$$

$$\theta = -\theta_s$$

$$Z\angle\theta = Zs\angle-\theta_s$$

Efficiency of Power Transfer:

With $Rs=R_L$ and $Xs= - X_L$ Substituting in (1)

$$P_{Lmax} = \frac{E^2_{TH}R}{(2R)^2} = \frac{E^2_{TH}}{4R}$$

and the power supplied is $Ps = \dfrac{E^2_{TH}\,2R}{(2R)^2} = \dfrac{E^2_{TH}}{2R}$

Then $\eta_{tra} = \dfrac{P_L}{Ps} = \dfrac{E^2_{TH}/4R}{E^2_{TH}/2R} = \dfrac{1}{2} = 50\%$

This means to transmit maximum power to the load 50% power generated is the loss. Such a low efficiency cannot be permitted in power systems involving large blocks of power where R_L is very large compared to Rs. Therefore constant voltage power systems are not designed to operate on the basis of maximum power transfer.

However in communication systems the power to be handled is small as these systems are low current circuits. Thus impedance matching is considerable factor in communication networks.

However between R & X if either R or X is restricted and between Z and θ if either |Z| or θ is restricted the conditions for Max P is stated as follows

Case (i) :- R of Z is varied keeping X constant with R only Variable, conditions for max power transfer is $(Rs+R)^2+(Xs+X)^2 - 2R(R_s+R)=0$

$$Rs^2+ R^2+ 2RsR+(Xs+X)^2-2RsR-2R^2=0$$

$$R^2= Rs^2+(Xs+X)^2$$

$$R= \sqrt{Rs^2 + (Xs + X)^2}$$

Case (ii):- If Z contains only R ie, x=0 then from the eqn derived above

$$R=|Zs|. \sqrt{Rs^2 + Xs^2}$$

Case (iii):- If |Z| is varied keeping θ constant then from (8) |Z|=|Zs|

Case (iv):- If |Z| is constant but θ is varied

Then from eqn (9) $(Z^2+Zs^2)\ \text{Sin}\ \theta =-2Z\ Zs\ \text{Sin}\theta s$

$$\text{Sin}\theta = \frac{-2ZZs}{(Z^2+Zs^2)}\ \text{Sin}\ \theta s$$

Then power transfer to load may be calculated by substituting for R and X for specified condition. For example

For case(ii) Pmax is given by

$$Pmax = \frac{E^2R}{(Rs+R)^2+(Xs+X)^2}$$

$$= \frac{E^2Zs}{(Rs+Zs)^2+Xs^2} = \frac{E^2Zs}{Rs^2+2RsZs+Zs^2+Xs^2}$$

$$= \frac{E^2}{2(Zs+Rs)} \quad (\text{ie } Rs^2+Xs^2= Zs^2)$$

Assignment questions:

1) State and explain superposition theorem and Norton's theorem
2) Obtain the Thevenin's equivalent of network shown in Fig. between terminals X and Y.

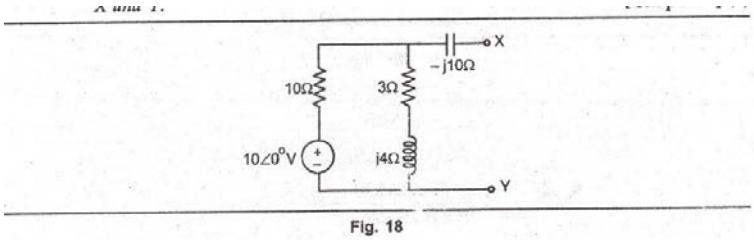


Fig. 18

3) Obtain the Thevenin's and Norton's equivalent circuits across terminals A and B for the circuit shown in Fig. 4.

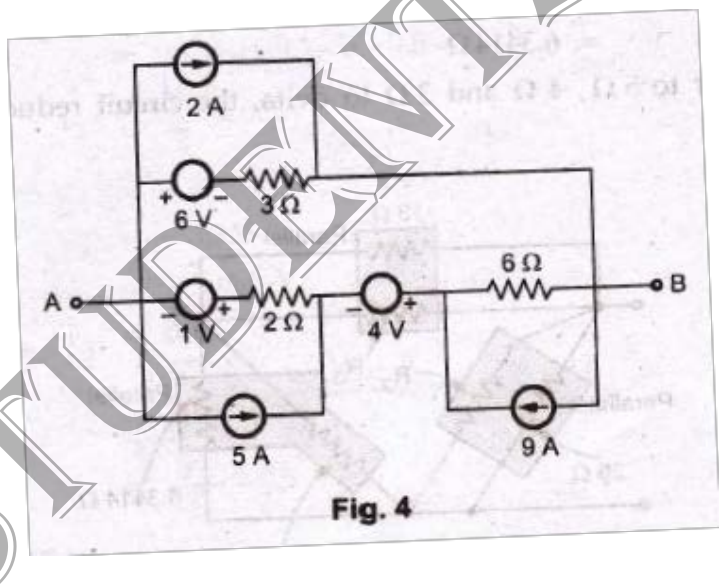


Fig. 4

4)) In the circuit of Fig. 7 obtain I by Thevenin's theorem

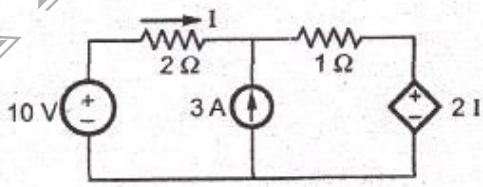


Fig. 7

5) state and prove Thevenin's theorem

6) Find Thevenin's equivalent circuit across AB using Millman's theorem and find the current through the load (5+j5) Ω shown in Fig. 8.

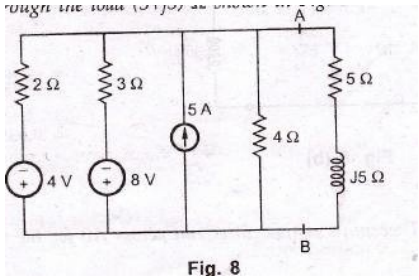


Fig. 8

7) Calculate Thevenin's equivalent circuit across AB for the network shown in fig. 9.

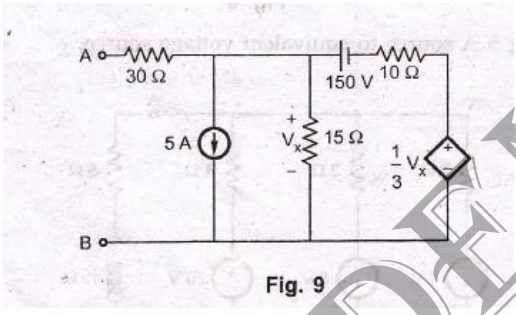


Fig. 9

Unit: 5: Resonant Circuits: Hrs: 06

Syllabus of unit :
Series and parallel resonance, frequency response of series and Parallel circuits, Q –factor, Bandwidth.

Recommended readings:

1. "Network Analysis", M. E. Van Valkenburg, PHI / Pearson Education

2. "Networks and systems", Roy Choudhury, 2 edition, New Age International Publications .

3. "Network theory ", Ganesh Rao.

4. "Network analysis" , Roy Choudry.

Resonant Circuits

Resonance is an important phenomenon which may occur in circuits containing both inductors and capacitors.

In a two terminal electrical network containing at least one inductor and one capacitor, we define resonance as the condition, which exists when the input impedance of the network is purely resistive. In other words a network is in resonance when the voltage and current at the network in put terminals are in phase.

Resonance condition is achieved either by keeping inductor and capacitor same and varying frequency or by keeping the frequency same and varying inductor and capacitor. Study of resonance is very useful in the area of communication. The ability of a radio receiver to select the correct frequency transmitted by a broad casting station and to eliminate frequencies from other stations is based on the principle of resonance.

The resonance circuits can be classified in to two categories

Series – Resonance Circuits.
Parallel – Resonance Circuits.

R L C

1.Series Resonance Circuit

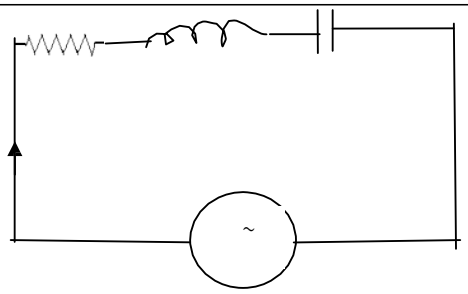
A series resonance circuit is one in which a coil and a capacitance are connected in series across an alternating voltage of varying frequency as shown in figure.

I

V

The response 'I' of the circuit is dependent on the impedance of the circuit,
Where Z= R +jXL - jXC
and I= V / z at any value of frequency

We have XL =27tfL ⇒ XL varies as f
and XC = 1 / 27tfC ⇒ XC varies inversely as f



In other words, by varying the frequency it is possible to reach a point where XL = XC . In that case Z = R and hence circuit will be under resonance. Hence the series A.C. circuit is to be under resonance, when inductive reactance of the circuit is equal to the capacitive reactance. The frequency at which the resonance occurs is called as resonant frequency (fr)

Expression for Resonant Frequency (fr)
At resonance XL = XC

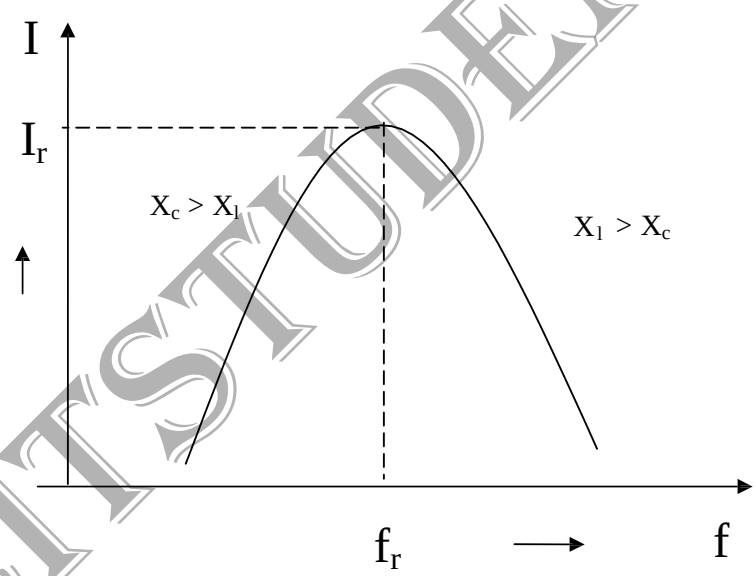
Salient Features of Resonant circuit

(*) At resonance XL = XC
(*) At resonance Z = R i.e. impedance is minimum and hence $I = \frac{V}{Z}$ is maximum
(*) The current at resonance (Ir) is in phase with the voltage
(*) The circuit power factor is unity
(*) Voltage across the capacitor is equal and opposite to the voltage across the inductor.

Frequency response of a series resonance circuit
For a R-L-C series circuit the current 'I' is given by

$$I = \frac{V}{R + j(XL - XC)}$$

At resonance XL = XC and hence the current at resonance (Ir) is given by Ir = V/R
At off resonance frequencies since the impedance of the circuit increases the current in the circuit will reduce. At frequencies f Where f> fr , the impedance is going to be more inductive. Similarly at frequencies f < fr the circuit impedance is going to be more capacitive. Thus the resonance curve will be as shown in figure.



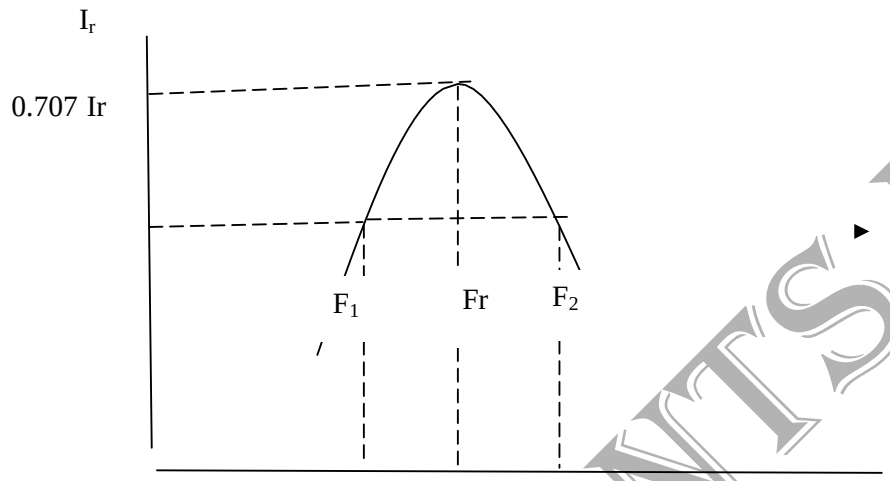
Qualify – factor (or Q – factor):

Another feature of a resonant circuit is the Q – rise of voltage across the resonating elements.
If V is the applied voltage across a series resonance circuit at resonance, $I_r = \frac{V}{R}$

Any circuit response, which is frequency dependent, has certain limitations. The output response during limited band of frequencies only will be in the useful range. If the out put power is equal to

or more than half of the maximum powerout put that band of frequencies is considered to be the useful band. If I r is the maximum current at resonance then
Power at resonance = Pmax = I2 r R

Consider the frequency response characterstic of a series resonant circuit as shown in figure



In the figure it is seen that there are two frequencies where the out put power is half of the maximum power. These frequencies are called as half power points f1 and f2

A frequency f1 which is below fr where power is half of maximum power is called as lower half power frequency (or lower cut – off frequency). Similarly frequency f2 which is above fr is called upper half power frequency (or upper cut-off frequency)

The band of frequencies between f2 and f1 are said to be useful band of frequencies since during these frequencies of operation the out put power in the circuit is more than half of the maximum power. Thus their band of frequencies is called as Bandwidth.

i.e Band width =B.W = f2 - f1

Selectivity :

Selectivity is a useful characteristic of the resonant circuit. Selectivity is defined as the ratio of band width to resonant frequency

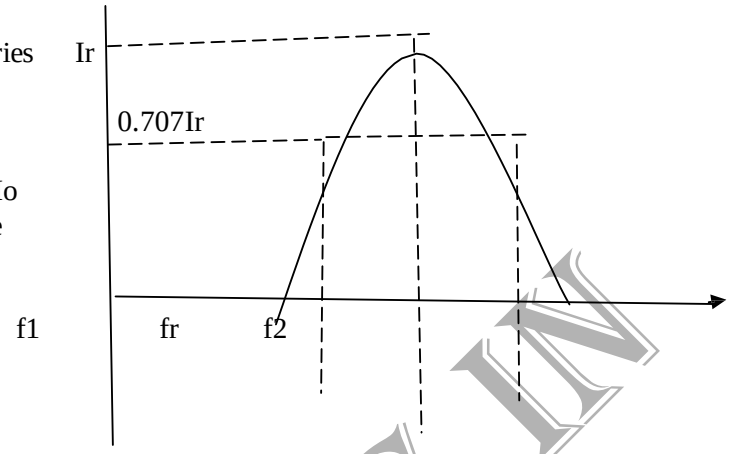
Selectivity = f2- f1
fr

It can be seen that selectivity is the reciprocal of Quality factor. Hence larger the value of Q Smaller will be the selectivity.

The Selectivity of a resonant circuit depends on how sharp the out put is contained with in limited band of frequencies. The circuit is said to be highly selective if the resonance curve falls very sharply at off resonant frequencies.

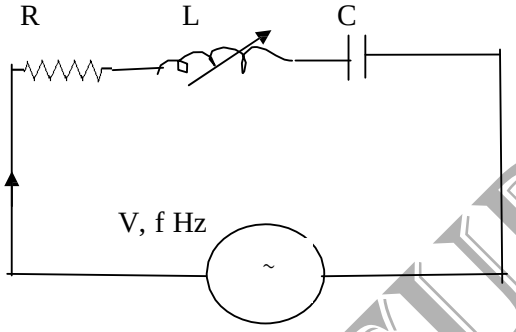
Relation between Resonant frequency and cut-off frequencies

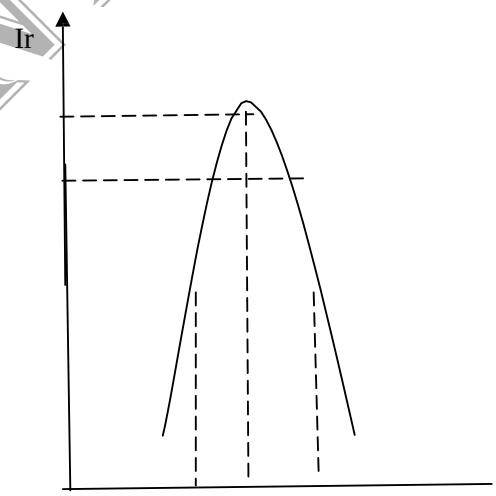
Let fr be the resonant frequency of a series resonant circuit consisting of R,L and C elements .From the Characteristic it is seen that at both half frequencies f2 and f1 the out put current is 0.707 Io which means that the magnitude of the impedance is same at these points.
At lower cut-off frequency f1



Resonance by varying Inductance

Resonance in RLC series circuit can also be obtained by varying resonating circuit elements . Let us consider a circuit where in inductance is varied as shown in figure.





Parallel Resonance

A parallel resonant circuit is one in which a coil and a capacitance are connected in parallel across a variable frequency A.C. Supply. The response of a parallel resonant circuit is somewhat different from that of a series resonant circuit.

Impedance at resonance

We know that at resonance the susceptive part of the admittance is zero.

Hence $Y_0 = \dfrac{R}{R^2 + m_0^2 L^2}$

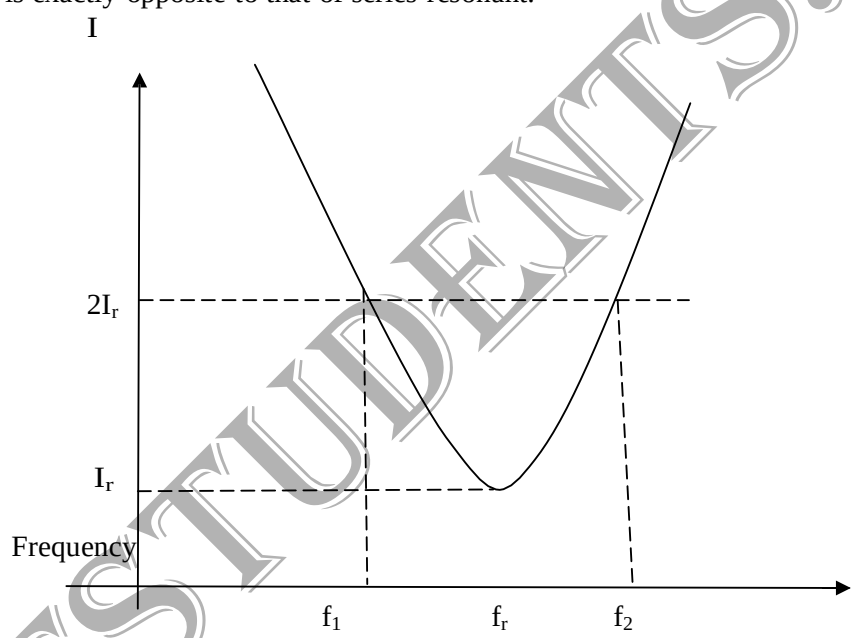
But $R^2 + m_0{}^2 L^2 = L/C$

So $Y_0 = RC/L$ or $Z_o = L/RC$

Where Z_o is called the dynamic resistance. when coil resistance R is small, dynamic resistance of the parallel circuit becomes high. Hence the current at resonance is minimum. Hence this type of circuit is called rejector circuit.

Frequency –response characterisitics

The frequency response curve of a parallel resonant circuit is as shown in the figure. We find that current is minimum at resonance. The half –power points are given by the points at which the current is $\sqrt{2}\, I_r$ .From the above characteristic it is clear that the characteristic is exactly opposite to that of series resonant.



Quality factor (Q-factor)

The quality factor of a parallel resonant circuit is defined as the current magnification

$$Q = \frac{\text{Current through capacitance at resonance}}{\text{Total Current at resonance}}$$

$= I_{C0} / I_0$

$= V / (1/ \omega_0 C) \; V / Z_0$

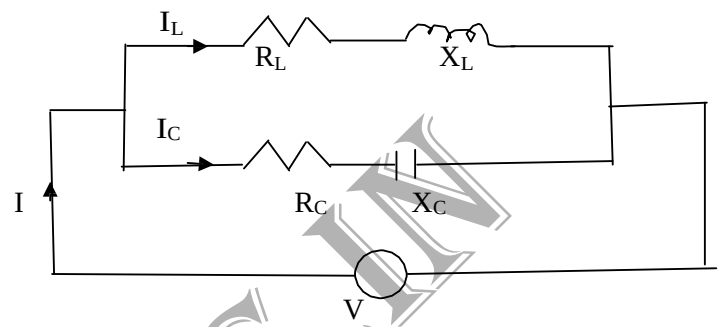
$= Z_0 \omega_0 C = (L / RC)\, \omega_0 C$

$= \omega_0 L / R$

Hence the expression for the Q- factor for both series and parallel resonant circuit are the same

Also Band width= f_0 / Q

II A coil and a Practical Capacitor in parallel



Consider a parallel resonant circuit in which the resistance of the capacitance is also considered

Impedance of the coil = $Z_L = R_L + j\,\omega L$

$$Y_L = 1 / Z_L = 1/ R_L + j\,\omega L$$
$$= R_L - j\,\omega L / R_L^2 + \omega^2 L^2$$

Impedance of the Capacitor = $Z_C = R_C - j / \omega C$

$$= \frac{R_C + j / \omega C}{Rc^2 + 1/\omega^2 C^2}$$

Therefore total admittance = $Y = Y_L + Y_C$

$$= \frac{(R_L - j\,\omega L / R_L^2 + \omega^2 L^2) + R_C + j / \omega C}{Rc^2 + 1/\omega^2 C^2}$$

At resonance the susceptance part of the total admittance is zero, which gives

$$\frac{1/\omega_0 C}{R_C^2 + 1/\omega_0^2 C^2} = \frac{\omega_0 L}{R_L^2 + \omega_0^2 L^2}$$

$$1/LC\,[R_L^2 + \omega_0^2 L^2] = \omega_0^2 [R_C^2 + 1/\omega_0^2 C^2]$$

$$\omega_0^2 (R_C^2 - L/C) = R_L^2 / LC - 1/C^2$$

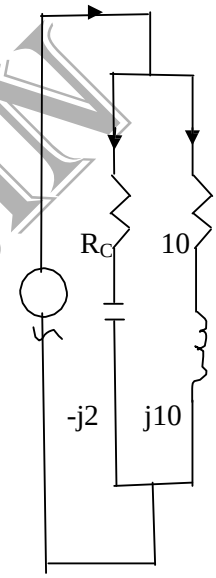
$$\omega_0^2 = \frac{1/LC(R_L^2 - L/C)}{(R_C^2 - L/C)}$$

$$\omega_0 = 1/\sqrt{LC}\,\sqrt{\frac{R_L^2 - L/C}{(R_C^2 - L/C)}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{(R_L^2 - L/C)}{(R_C^2 - L/C)}}$$

At Resonance the admittance is purely conductive given by

$$Y_0 = \frac{R_L}{R_L^2 + \omega_0^2 L^2} + \frac{R_C}{R_C^2 + 1/\omega_0^2 C^2}$$



<u>Example 1</u> : Determine the value of R_C in the Shown in figure to yield Resonance

Solution: Let Z_L be the impedance of the inductive branch

then $Z_L = 10 + j\,10$

$Y_L = 1/(10 + j\,10)$

$= \frac{10 - j\,10}{10^2 + 10^2} = \frac{10 - j\,20}{200}$

Let Z_C be the impedance of the capacitive branch then

$Z_C = R_C - j\,2$

$$Y_C = \frac{1}{R_C - j\,2} = \frac{R_C - j\,2}{R_C^2 + 4}$$

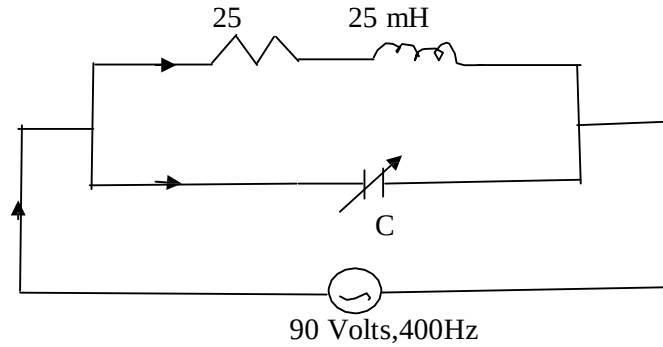
Total admittance of the circuit = $Y = Y_L + Y_C$

For the circuit under Resonance the Susceptance part is zero

$$(2/R_C^2 + 4) - (10/200) = 0$$

$R_C^2 = 36$

$R_C = 6$ ohms Answer



Example 2: An Impedance coil of 25 ohms Resistance and 25 mH inductance is connected in parallel with a variable capacitor. For what value of Capacitor will the circuit resonate.If 90 volts,400 Hz source is used, what will be the line Current under these conditions

Solution: $\omega_0 = 2\pi f_0 = 2\pi(400)$

$$m_0^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$6.316 \times 10^6 = \frac{1}{} - \frac{R^2}{}$$

LC L 2

$$\frac{1}{LC} = m_0^{\,2} + \frac{R^2}{L^2}$$

$= 6.316 \times 10^6 + 25^2 / (25 \times 10^{-3})^2$

$= 7.316 \times 10^6$

C= 5.467 μF

$Z_0 = L/RC = (25 \times 10^{-3}) / 25 \times 5.467 \times 10^{-6}$

= 182.89 ohms

$I_0 = V_0 / Z_0 = 90 / 182.89 = 0.492$ ampere

Assignment questions:

1) Explain parallel resonance ? Derive the condition for parallel resonance when RL connected parallel to RC .
2) Show that resonant frequency of series resonance circuit is equal to the geometric mean of two half power frequencies.
3) In the circuit given below in Fig. 21, an inductance of 0.1 H having a Q of 5 is in parallel with a capacitor. Determine the value of capacitance and coil resistance at resonant frequency of 500 rad/sac.

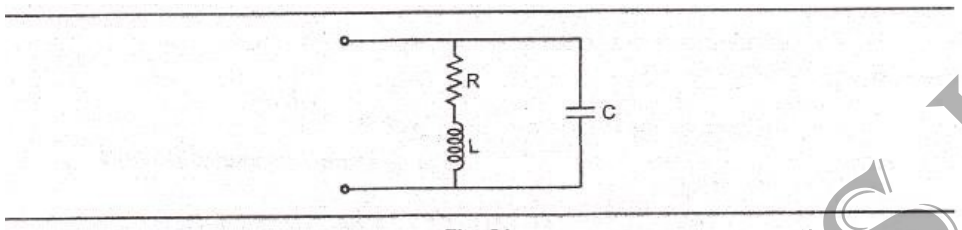


Fig. 21

4) In the circuit shown in Fig. 22, determine the complete solution for the current when the switch S is closed at t = 0. Applied voltage is V (t) = 400 cos 500+ $\frac{\pi}{4}$
Resistance R = 15Ω, inductance L = 0.2H and capacitance C = 3 μF.

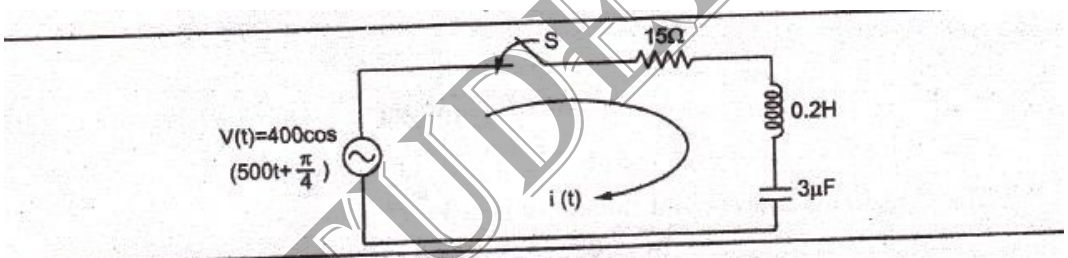


Fig. 22

5) In the circuit shown in fig. 23 the switch S is moved from a to b at t = 0. Find values of i, $\frac{di}{dt}$, $\frac{di^2}{dt^2}$ at t = 0^+ if R = 1Ω, L = 1H, c= 0.1 μF and V=100 V.
Assume steadily state is achieved when K is at 'a'.

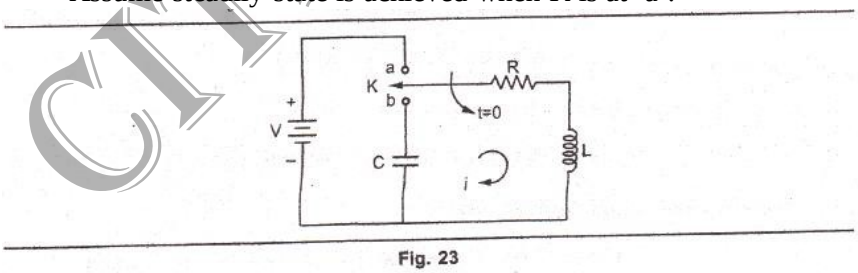


Fig. 23

6) A series resonant circuit includes 1 ⅡF capacitor and a resistance of 16Ω? If the band Width is a 500 rad/sec, determine i) ω_r ii) Q and iii) L

7) A two branch antiresonant circuit contains L = 0.4H and C = 40ⅡF. Resonance is to be achieved by variation of RL and R_C. Calculate the resonance frequency for the following cases:

i) R_L = 120Ω, R_C = 80Ω

ii) RL = 80Q, Rc = 0

iii) RL = Rc = lOOQ

Unit: 6 Transient behavior and initial conditions Hrs: 07

Syllabus of unit :

Behavior of circuit elements under switching condition and their Representation, evaluation of initial and final conditions in RL, RC and RLC circuits for AC and DC excitations.

Recommended readings:

1. "Network Analysis", M. E. Van Valkenburg, PHI / Pearson Education

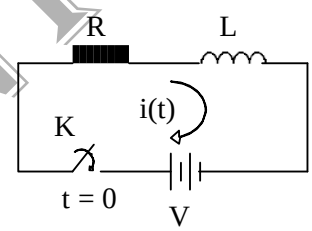
2. "Networks and systems", Roy Choudhury, 2 edition, New Age International Publications .

3. "Network theory ", Ganesh Rao.

4. "Network analysis" , Roy Choudry.

Electrical circuits are connected to supply by closing the switch and disconnected from the supply by opening the switch. This switching operation will change the current and voltage in the device. A purely resistive device will allow instantaneous change in current and voltage.
An inductive device will not allow sudden change in current or delay the change in current. A capacitive device will not allow sudden change in voltage or delay the change in voltage. Hence when switching operation is performed in inductive or capacitive device the current and voltage in the device will take a certain time to change from preswitching value to steady value after switching. This study of switching condition in network is called transient analysis. The state (or condition) of the current from the instant of switching to attainment of steady state is called transient state or transient. The current and voltage of circuit elements during transient period is called transient response.
The transient may also occur due to variation in circuit elements. Transient analysis is an useful tool in electrical engineering for analysis of switching conditions in Circuit breakers, Relays, Generators etc.
It is also useful for the analysis of faulty conditions in electrical devices. Transient analysis is also useful for analyzing switching Conditions in analog and digital Electronic devices.

R-L Series circuit transient:



Consider The R-L series circuit shown in the fig. Switch K is closed at t=0. Referring to the circuit, balance equation using Kirchoff's law can be written as

$$V(t) = Ri(t) + L\frac{di(t)}{dt}$$

Taking Laplace Transform we get

$$\frac{V(s)}{s} = I(s) \times R + L\{SI(s) - i(0)\}$$

Assuming there is no stored energy in the inductor
I(0)=0

$$\frac{V(s)}{s} = RI(s) + LSI(s)$$

$$I(s) = \frac{V(s)}{S(R+SL)} = \frac{V(s)}{L}\left[\frac{1}{S\left(S+\frac{R}{L}\right)}\right]$$

$$I(s) = \frac{A}{S} + \frac{B}{S+\frac{R}{L}}$$

$$A\left(S+\frac{R}{L}\right) + BS = \frac{V(s)}{L}$$

Put s=0 $$A\left(\frac{R}{L}\right) = \frac{V(s)}{L}$$

$$A = \frac{V(s)}{R}$$

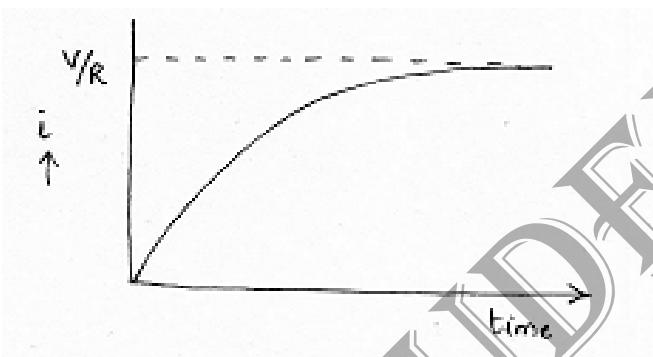
put $$S = \frac{-R}{L} \quad B\left(\frac{-R}{L}\right) = \frac{V(s)}{L}$$

$$B = \frac{-V(s)}{R}$$

Therefore $$I(s) = \frac{V(s)}{R}\left[\frac{1}{S} - \frac{1}{S + \frac{R}{L}}\right]$$

Taking inverse Laplace we get

$$i(t) = \frac{V}{R}\left[1 - e^{-\left(\frac{R}{L}\right)t}\right]$$



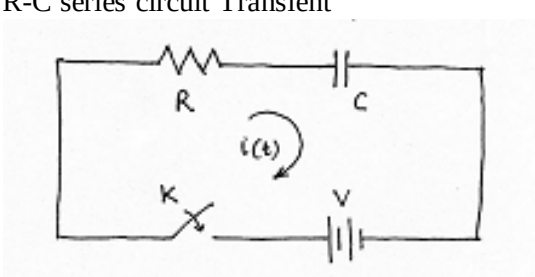
The equation clearly indicates transient nature of current, which is also shown in figure.

Where $\frac{L}{R} =$ Tune constant of the circuit, which is denoted by Z given in seconds.

Hence $$i(t) = \frac{V}{R}\left[1 - e^{\frac{-Z}{t}}\right]$$

Putting t=z we get i(z) = 0.632 $\frac{V}{R}$ Where $\frac{V}{R}$ = steady state current. Hence Time constant for an R-L series current circuit is defined as the time taken by the circuit to reach 63.2% of its final steady value.

R-C series circuit Transient



Consider the RC circuit shown. Let the switch be closed at t=0.

Writing the balance equation using Kirchoff's voltage law ,

$$v(t) = iR + \frac{1}{c}\int i\,dt$$

Taking Laplace transform, we get

$$\frac{V(s)}{S} = I(s)R + \frac{1}{c}\left[\frac{I(s)}{s} + \frac{Q(0^-)}{s}\right]$$

Let us assume that there is no stored energy in the circuit.

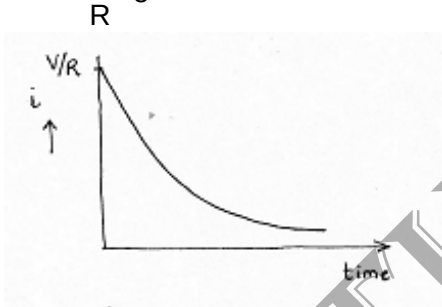
Hence $Q(0^+)$ =0

$$\frac{V(s)}{s} = I(s)R + \frac{I(s)}{CS} = I(s)\left[R + \frac{1}{CS}\right]$$

$$I(s) = \frac{V(s)}{R}\left(\frac{1}{S + \frac{1}{RC}}\right)$$

Taking Laplace inverse we get

$$i(t) = \frac{V(t)}{R}e^{-\left(\frac{1}{RC}\right)t}$$



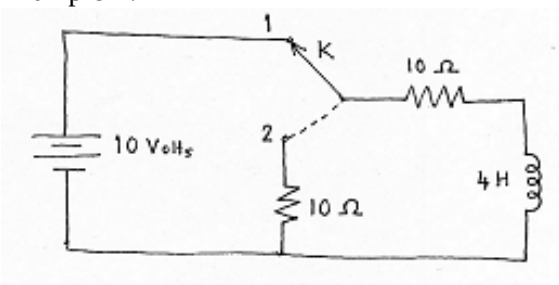
The sketch of transient current is shown in figure Where $\frac{1}{RC} = \tau$ the time constant of the circuit.

Putting $\tau = \frac{1}{RC}$ in the current equation we get

i(z) = 0.367 $\frac{V}{R}$

Hence time constant of RC series current can be defined as the time taken by current transient to fall to 36.7% of its initial value.

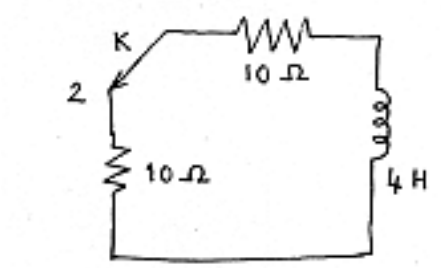
Example 1:



In the circuit shown in figure the switch 'K' is moved from position 1 to position 2 at time t = 0. The steady state current having been previously established in R-L circuit. Find the current i(t) after switching.

Solution:

From the given data the circuit is under steady state when switch K is in position 1 under steady state condition inductance is a short and hence i(0) = $\frac{10}{10}$ = 1 Amp.



When the circuit is switched to position 2, this 1 Amp current constituted the stored energy in the coil.

Writing the balance equation for position 2 we get $20i + 4\frac{di}{dt} = 0$

Taking Laplace transformation

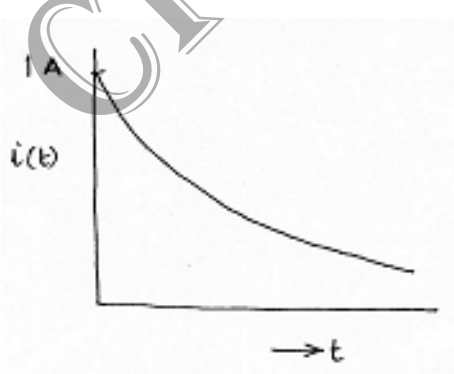
$$20I(s) + 4[s\,I(s) - i(0)] = 0$$

$$20I(s) + 4[s\,I(s) - 1]$$

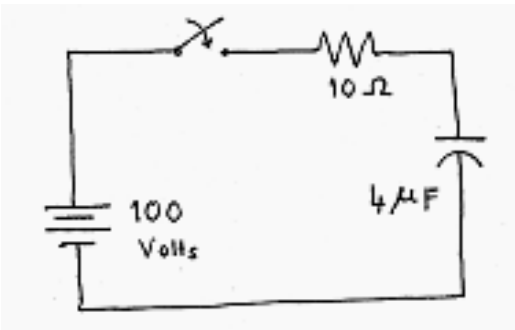
$$I(s) = \frac{4}{4(s+5)} = \frac{1}{s+5}$$

taking inverse Laplace we get

$$i(t) = e^{-5t}$$



Example 2:



A series R-C circuit is shown in figure. The capacitor has an initial charge of 800μCoulombs on its plates, at the time the switch is closed. Find the resulting current transient.

Solution: From the data given q(0) = 800×10^{-6} C

Writing the balance equation we get

$$100 = 10i(t) + \frac{1}{4 \times 10^{-6}} \int i(t)dt$$

Taking Laplace transformation

$$\frac{100}{S} = 10I(s) + \frac{1}{4 \times 10^{-6} S}\left[I(s) - Q(0)\right]$$

$$I(s)\left[10 + \frac{10^6}{4S}\right] = \frac{100}{S} + \frac{800 \times 10^{-6}}{4 \times 10^{-6} S}$$

$$= \frac{100 + 200}{5}$$

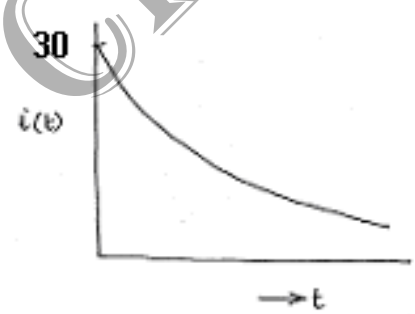
$$I(s)\left[\frac{40S + 10^6}{4S}\right] = \frac{30}{S}$$

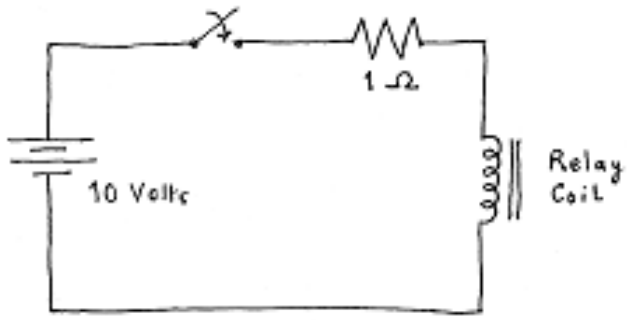
$$I(s) = \frac{1200}{40S + 10^6} = \frac{1200}{40\left(S + \frac{10^6}{40}\right)}$$

$$I(s) = \frac{30}{S + 25000}$$

Taking Inverse Laplace we get

$$i(t) = 30e^{-25000t}$$





Example3:

For the circuit shown in figure the relay coil is adjusted to operate at a current of 5 Amps. Switch K is closed at t = 0 and the relay is found to operate at t = 0.347 seconds. Find the value of inductance 'L' of the relay.

Soln: Writing the balance equation for the relay circuit

$$V(t) = Ri(t) + L\frac{di}{dt}$$

Applying Laplace transformation

$$\frac{V(s)}{S} = RI(S) + LS[I(S) - i(0)]$$

Since there is no mention of initial current in the coil i(0) =0

Hence $\frac{10}{S} = I(s) + I(s)LS$

$$I(S)\{SL + 1\} = \frac{10}{S}$$

$$I(s) = \frac{10}{S(1+SL)} = \frac{\frac{10}{L}}{S\left(S+\frac{1}{L}\right)} = \frac{A}{S} + \frac{B}{\left(S+\frac{1}{L}\right)}$$

$$\frac{10}{L} = A\left(S+\frac{1}{L}\right) + BS$$

A=10 B= -10

$$I(S) = 10 - \frac{10}{S+\frac{1}{L}}$$

Taking Inverse Laplace we get

$$i(t) = 10 - 10e^{-\frac{t}{L}}$$

The relay operates at t = 0.347 seconds when the current value reaches 5A. Hence

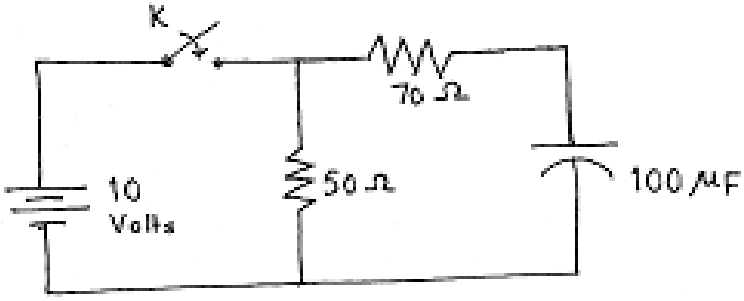
$$5 = 10 - 10e^{\frac{-0.347}{L}}$$

$$10e^{\frac{-0.347}{L}} = 10 - 5 = 5$$

$$e^{\frac{-0.347}{L}} = 5$$

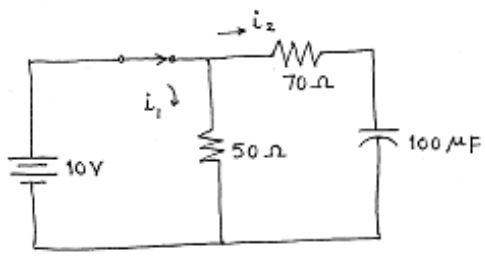
Solving the equation we get L=0.5H

Example 4:



In figure the switch 'K' is closed. Find the time when the current in the circuitry reaches to 500 mA

Soln: When the switch is closed Vc (0) = 0
When the switch is closed at t = 0



I1 (t)×50 = 10

$$I_2(t) \times 70 + \frac{1}{100 \times 10^{-6} \int i_2 dt} = 10$$

Taking Laplace for both the equations

$$I_1(S) = \frac{10}{50S} = \frac{0.2}{5} \text{ - - - - - - - (1)}$$

$$I_2(S) \times 70 + \frac{1}{c} \frac{I_2(s)}{S} = \frac{10}{5}$$

$$70 I_2(s) + \frac{I_2(S)}{100 \times 10^{-6} s} = \frac{10}{5}$$

$$I_2(S) = \frac{10}{70S + 10^4} = \frac{1}{7s + 10^3} = \frac{1}{7}, \frac{1}{S + 142.86} (2)$$

Taking inverse Laplace for equation (1) and (2)
I1 (t) =0.2 A

$$I_2(t) = \frac{1}{7} e^{-142.86t}$$

Total current from the battery i(t) =I1 + I2

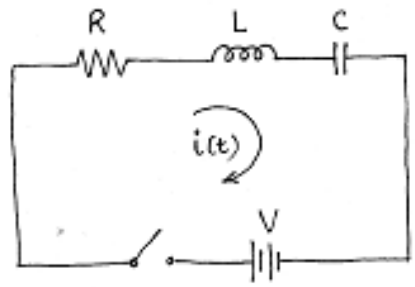
$$i(t) = 0.2 + \frac{1}{7} e^{-142.86t}$$

when this current reaches 500 mA

$$500 \times 10^{-3} = 0.2 + \frac{1}{7} e^{-142.86t}$$

Solving we get t = 5.19 × 10-3 Seconds.

R-L-C Series Transient circuit:



Assuming zero initial conditions when switch K is closed the balanced equation is given by

$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int idt$$

Taking Laplace transformation we get

$$\frac{V(s)}{s} = I(s)R + LSI(s) + \frac{I(s)}{CS}$$

$$= I(s)\left[R + SL + \frac{1}{CS}\right]$$

$$I(s) = \frac{V(s)}{S(R + SL + \frac{1}{CS})} = \frac{\frac{V(s)}{L}}{S^2 + \frac{R}{L}S + \frac{1}{LC}}$$

The time response of the circuit depends on the poles or roots of the characteristic equation

$$S^2 + S\frac{R}{L} + \frac{1}{LC} = 0$$

Roots of the characteristic equation are given by

$$S_1, S_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4 \times \frac{1}{LC}}}{2}$$

$$S_1, S_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Intial conditions:

The reason for studying initial and final conditions in a network is to evaluate the arbitrary constants that appear in the general solution of the differential equations written for the network.

In this chapter we concentrate on finding the change in selected variables in a circuit when a switch is thrown from open to closed or vice versa position. Please note that t = 0 indicates the time of throwing the switch

t = 0- indicates time immediately before throwing the switch and

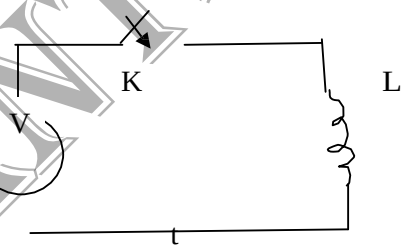
t =0+ indicates time immediately after throwing the switch.
We are very much interested in the change in currents and voltages of energy storage elements (inductor and capacitor) after the switch is thrown since these variables along with the sources will dictate the circuit behavior for t > 0.

Initial conditions in a network depend on the past history of the circuit (before t= 0-) and structure of the network at t = 0+.Past history will show up in the form of capacitor voltages and inductor currents.

Initial and final conditions in elements

The resistor:
The cause effect relation for the ideal resistor is given by v = Ri. From this equation we find that the current through a resistor will change instantaneously, if the voltage changes instantaneously.Similarly voltage will change instantaneously if current changes instantaneously.

The inductor :



Initial condition

The switch is closed at t= 0

The expression for current through the inductor is given by $i(t) = \frac{1}{L} \int_{-\infty}^{t} v\,dt$

$$= \frac{1}{L}\int_{-\infty}^{0-} v\,dt + \frac{1}{L}\int_{0-}^{t} v\,dt$$

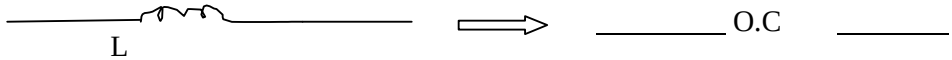
$$= i(0-) + \frac{1}{L}\int_{0-}^{t} v\,dt$$

Putting t = 0+

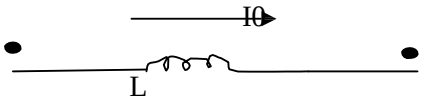
$$i(0+) = i(0-) + \frac{1}{L}\int_{0-}^{0+} v\,dz$$

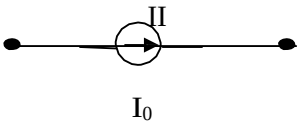
$$i(0+) = i(0-)$$

The above equation indicates that the current in an inductor can not change instantaneously. Hence if i (0-) =0, then i(0+) = 0. This means that at t = 0+ inductor will act as an open circuit, independent of voltage across the terminals.



If i (0-) = I0 (i. e. if a residual current is present) then i (0+) = I0 , meaning that an inductor at t = 0+ can be thought of as a current source of I0 which is as shown



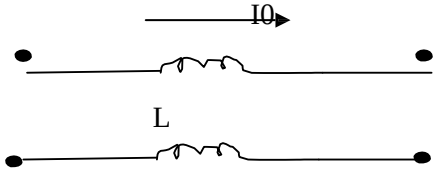


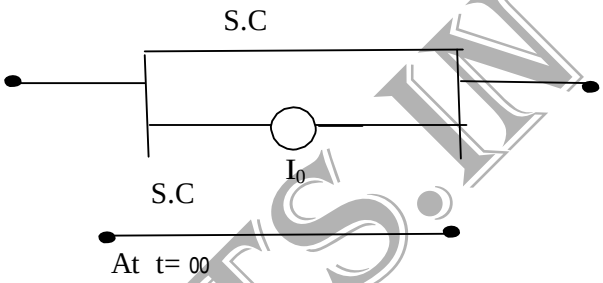
Final (or steady state) condition

The final –condition equivalent circuit of an inductor is derived from the basic relation ship V = L di/ dt

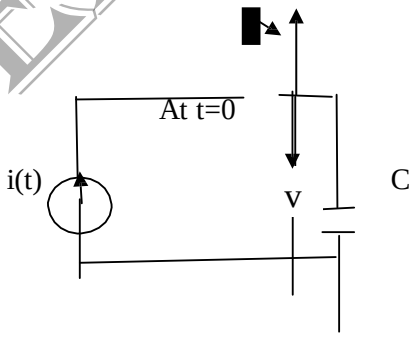
Under steady state condition di = 0 which means v = o and hence L acts as a short

at t = 00 (final or steady state)





The capacitor



The switch is closed at t = 0 . The expression

For voltage across the capacitor is given by

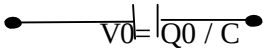
$$v = 1/C \int_{-00}^{t} i\,dt$$

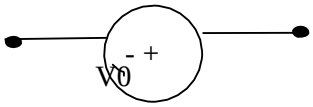
$$v(t) = 1/C \int_{-00}^{0-} i\,dt + 1/C \int_{0-}^{t} i\,dt$$

Putting t= 0+

$$v(0+) = v(0-) + 1/C \int_{0-}^{0+} i\,dt$$

V(0+) = V (0-) which means that the voltage across the capacitor can not change instantaneously. If V(o-) = o then V (o+) = o indicating that the Capacitor acts as a short at t=0+



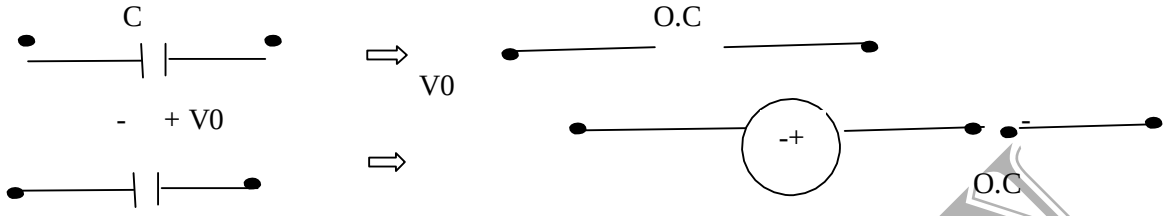


Final (or steady state) condition

The final-condition equivalent network is determined from the basic relationship

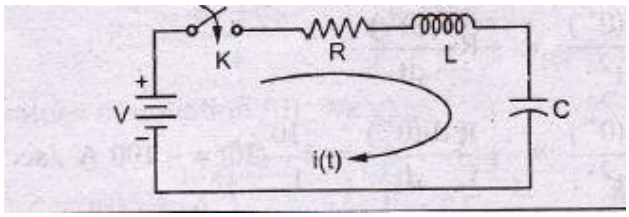
$$i = C\, dv/dt$$

Under steady state condition $dv / dt = 0$ which means at $t = \infty$ the Capacitor acts as a open circuit.

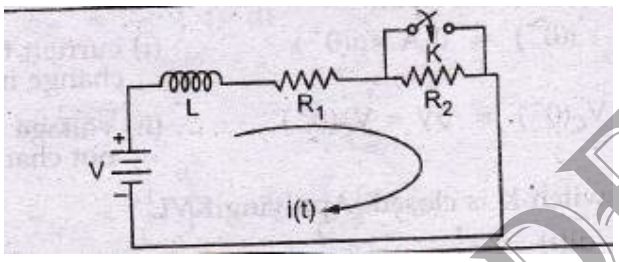


Assignment questions:

1) Establish the procedure for evaluating initial conditions with suitable examples.
2)) In the circuit shown in Fig. 20 V=10V, R= 10Ω, L=1H, C= 10μF, and V_C=0 . find i(0+), $\frac{di}{dt}$(0+) and $\frac{d^2i}{dt^2}$ (0+) it switch K is closed at t=0.



3)) Find i(t) for the following network shown in Fig. 21 if if the switch 'K' is opened at t= 0, before that the circuit has attained steady state condition .



4) R = 1Ω, L = 1H and C = 1/2F are in series with a switch across C 2V is applied to the circuit. At t = 0^- the switch is in closed position. At t = 0 the switch is opened. Find at t = 0^+ , the voltage across the switch, its first and second derivatives.

5) A coil of R = 1000Ω and L = 1 H is connected to a d.c. voltage of 100V through a changeover switch. At t = , the switch connects a capacitor of C = 0.1μF in series with the coil, excluding the voltage. Solve for i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ in the coil all at t = 0^+.

6) Use initial and final value theorems, where they apply, to find f(0) and f(∞) for the following :

i) $F(s) = \frac{S^3+7S^2+5}{s(S^3+3S^2+4S+2)}$

ii) $F(s) = \frac{e^{2s}(s+2)}{s^2+5}$

iii) $F(s) = \frac{s(s+4)(s+8)}{(s+1)(s+6)}$

7) Why do we need to study initial condition? Write the equivalent from of the elements in terms of the initial condition of the element

8) A parallel R-L circuit is energized by a current source of 1 A. the switch across the source is opened at t=0. Solve for V, D_v and D^2v all at $t=0^+$ if R=100Ω and L=1H.

9) Determine the Thevenin's equivalent $V_{ab}(S)$ and $Z_{ab}(S)$ for the network on Fig.12 for zero initial conditions.

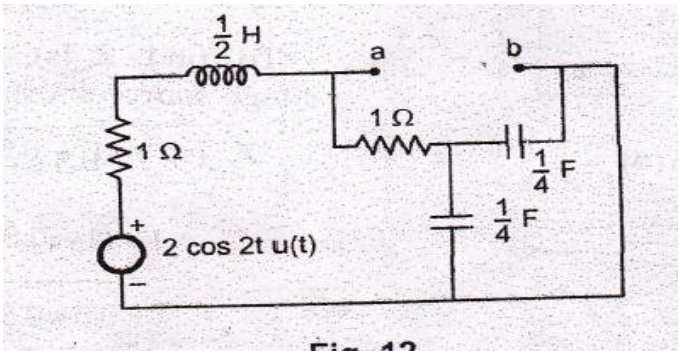


Fig. 12

10) For the circuit shown in Fig. 15 the switch is opened at t=0. If L = $\frac{1}{2}$H, G=1mho, C =1F and V=1v, find the node voltages v1(t) and v2(t)

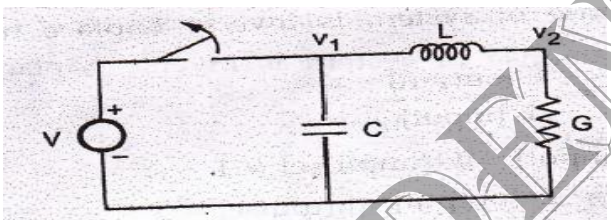


Fig. 15

Unit: 7 Laplace Transformation & Applications : Hrs: 07

Syllabus of unit :

Solution of networks, step, ramp and impulse responses, waveform Synthesis

Recommended readings:

1. "Network Analysis", M. E. Van Valkenburg, PHI / Pearson Education

2. "Networks and systems", Roy Choudhury, 2 edition, New Age International Publications .

3. "Network theory ", Ganesh Rao.

4. "Network analysis" , Roy Choudry.

LAPLACE TRANSFORMATION:

Laplace transform is a very useful and powerful tool in circuit analysis.Integro-differential equations Can be transformed in to algebraic equations using the technique of Laplace transformation and complete solution involviong both natural response and forced response is obtained in one step

Definition of Laplace Transform :

Let f(t) be a function of time.Assuming the value of function to be zero for t<0,the Laplactransform of f(t) is given as

$$L[f(t)] = F(S) = \int_0^{\infty} f(t)\, e^{-St}\, dt$$

f(t) is a function in time-domain and F(S) is a function in complex frequency domain. Complex frequency 'S' is given by S= ¬ +jω

From the above it is obvious that Laplace transformation changes a function in time domain into a function in frequency domain.

Important properties of Laplace transform

Linearity Property:

If L { f1(t) }= F1(S)

And L{ f2(t) }=F2(S)

Then L {a1 f1(t) }+ a2 f2(t)}= a1 F1(S)+ a2 F2(S)

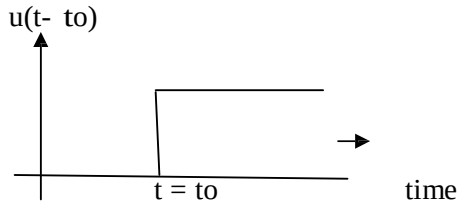
Proof: $L\{a_1 f_1(t) + a_2 f_2(t)\} = \int_0^{\infty} \{a_1 f_1(t) + a_2 f_2(t)\} e^{-St}\, dt$

$$= a_1 \int_0^{\infty} f_1(t)\, e^{-St}\, dt + a_2 \int_0^{\infty} f_2(t)\, e^{-St}\, dt$$

$$= a_1 F_1(S) + a_2 F_2(S)$$

Time-Shifting Property:

If L{ x(t) } = X(S) then for any real number t0
L{ x(t- t0) u(t- t0)}= e-to S X(S)



Proof: Let $L\{x(t-t_o)\, u(t-t_o)\} = \int_0^{\infty} x(t-t_0)\, u(t-t_0)\} e^{-St}\, dt$

Since u(t- to)= 1 }t>t0
0 }t<t0

We get $L[x(t-t_0)\,u(t-t_o)] = \int_{t_o}^{\infty} x(t-t_0)e^{-St}\,dt$

Let $t = t_0 + \tau$ $\quad dt = d\tau$

As $t \to t_0$ $\quad \tau \to 0$

As $t \to \infty$ $\quad \tau \to \infty$

Hence we get $L\{x(t-t_0)\,u(t-t_o)\} = \int_0^{\infty} x(\tau)\,e^{-S(\tau + t_0)}\,d\tau$

$= e^{-St_o} \int_0^{\infty} x(\tau)\,e^{-S\tau}\,d\tau$

$= X(S)\,e^{-St_o}$

Frequency-domain shifting property

If $L\{x(t)\} = X(S)$ then

$L\{e^{S_o t}\,x(t)\} = X(S-S_o)$

Proof: $L\{e^{S_o t}\,x(t)\} = \int_0^{\infty} e^{S_o t}\,x(t)\,e^{-St}\,dt$

$= \int_0^{\infty} x(t)\,e^{-(S-S_o)t}$

$= X(S-S_0)$

4. Time-Scaling Property

If $L[x(t)] = X(S)$ then

$L[x(at)] = 1/a\,X(S/a)$

Proof: $L[x(at)] = \int_0^{\infty} x(at)\,e^{-St}\,dt$

Put $at = \tau$

$a\,dt = d\tau$

Hence $L[x(at)] = \int_0^{\infty} 1/a\,x(\tau)\,e^{-S\tau/a}\,d\tau$

Hence $L[x(at)] = 1/a \int_0^{\infty} x(\tau)\,e^{-S\tau/a}\,d\tau$

$L[x(at)] = 1/a\,X(S/a)$

5. Time-Differentiation Property

If $L[x(t)] = X(S)$

$L[dx/dt] = S\,X(S) - x(0)$

Proof: Let $y(t) = dx/dt$

Then $L[y(t)] = Y(S) = \int_0^{\infty} y(t)\, e^{-St}\, dt$

$$= \int_0^{\infty} (dx/dt)\, e^{-St}\, dt$$

$$= \left[e^{-st} x(t) \right]_0^{\infty} - \int_0^{\infty} x(t) \{ -se^{-St} \} dt$$

$$= [x(\infty) - x(0)] + s \int_0^{\infty} x(t)\, e^{-St}\, dt$$

$$Y(S) = SX(S) - x(0)$$

i.e. $L[dx/dt] = S\,X(S) - x(0)$

6. Time-integration Property

For a Causal signal $x(t)$, if $y(t) = \int_0^{\infty} x(\tau)\, d\tau$ Where 'τ' is a dummy variable of 't'

Then $L[y(t)] = Y(S) = \frac{X(S)}{S}$

Proof $L\{x(t)\} = X(S) = \int_0^{\infty} x(t)\, e^{-St}\, dt$

Dividing both sides by S yields

$$\frac{X(S)}{S} = \frac{\int_0^{\infty} x(t)\, e^{-St}\, dt}{S}$$

$$= \frac{e^{-St}}{S} \int_0^{\infty} x(t) dt - \int_0^{\infty} \left[\frac{x(t)}{S} e^{-St}(-s)\, dt \right]$$

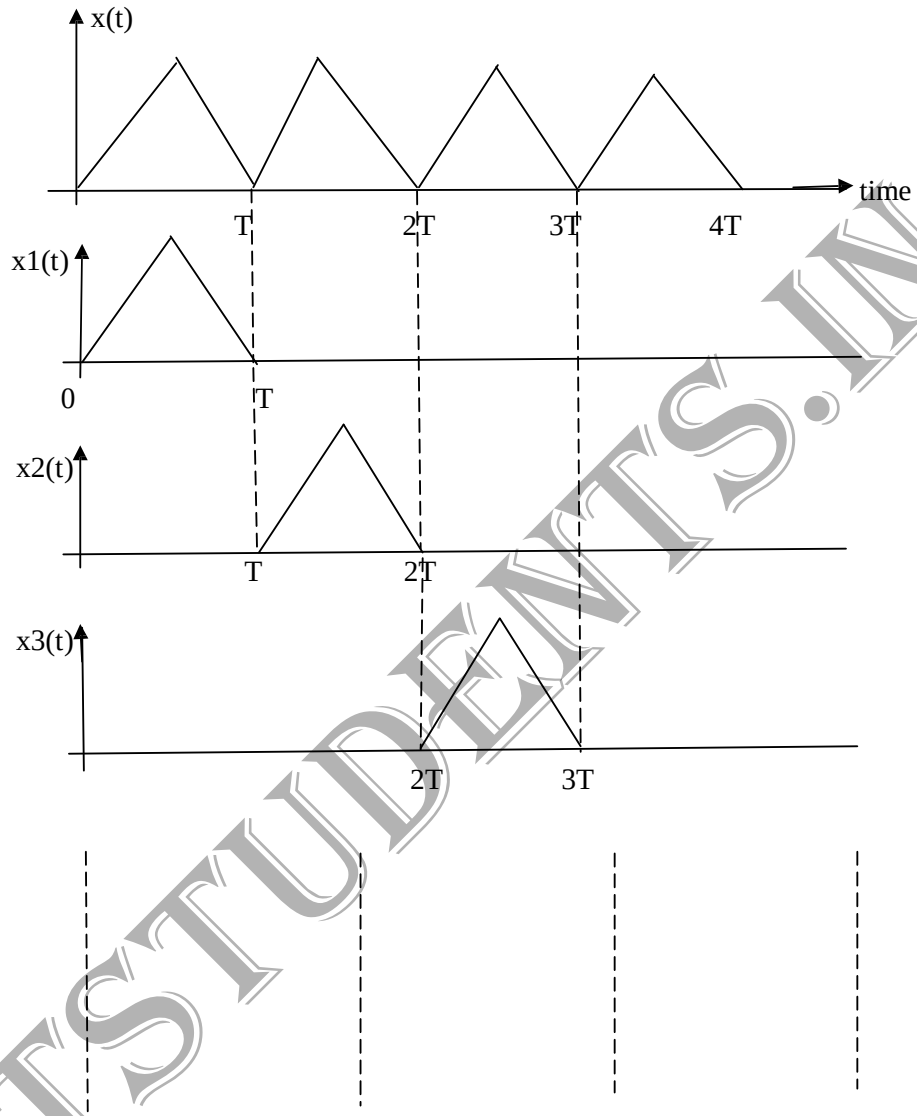
$$= \frac{e^{-St}}{S} y(t) - \int_0^{\infty} \frac{y(t)}{S} e^{-St}(-s)\, dt$$

$$= \frac{e^{-St}}{S} y(t) - Y(S)$$

$\frac{X(s)}{S} = Y(S)$ as $\frac{e^{-St}}{S} y(t) = 0$ at $t = \infty$

Therefore $Y(S) = \frac{X(S)}{S}$

7. Time-Periodicity Property



Let us consider a Function x(t) that is periodic as shown in figure.The function x(t) can be represented as the sum of time-shifted functions as shown in figure.

Hence x(t) = x1(t) + x2(t) + x3(t) + ……

Where x2(t) = x1(t- T) u(t-T)

x3(t) = x1(t- 2T) u(t-2T)

……

…… and so on

Hence x(t) = x1(t) + x1(t- T) u(t-T)+ x1(t- 2T) u(t-2T) + ……..

Where x1(t) is the waveform described over the first period of x(t).

Taking Laplace transformation on both sides of the above equation we get

X(s) = X1(s) + X1(s) e-TS + X1(s) e-2TS + X1(s) e-3TS + ……

$$= X1(s)[1+ e-TS + e-2TS + e-3TS + \ldots\ldots\ldots\ldots\ldots\ldots]$$

But $1+ a2 + a3 + a4 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots = \frac{1}{1-a}$ For $a< 1$

Hence we get $X(S) = X1(S) \left(\frac{1}{1- e-TS} \right)$

Initial –value Theorem:

The Initial -value theorem allows us to find the initial value x(0) directly from the Laplace Transform X(S).If x (t) is a casual signal, then $x(0) = \lim_{S \to 00} S\,X(S)$

Proof: WE have $L\left\{ \frac{dx(t)}{dt} \right\} = S\,X(S) - x(0)$

$$\int_0^{00} (dx/dt)\, e-St\, dt = S\,X(S) - x(0)$$

Taking Limit on both sides as $S \to 00$,we get

$$\lim_{S \to 00} \int_0^{00} (dx/dt)\, e-St\, dt = \lim_{S \to 00} S\,X(S) - x(0)$$

Hence $0 = \lim_{S \to 00} S\,X(S) - x(0)$ or $\lim_{S \to 00} S\,X(S) = x(0)$

Final Value theorem

The final value theorem allows us to find the value of x(00) directly from its Laplace Transformation X(S)

If x(t) is a casual signal, $\lim_{t \to 00} x(t) = \lim_{S \to 0} S\,X(S)$

Proof:

We have $L\ \frac{dx(t)}{dt} = S\,X(S) - x(0)$

$$\int_0^{00} (dx/dt)\, e-St\, dt = S\,X(S) - x(0)$$

Taking the limits $S \to 0$ on both sides we get,

00

$$\lim_{S \to 0} [\, S\,X(S) - x(t)] = \lim_{S \to 0} (dx/dt)\, e^{-St}\, dt$$

$$\int_0^{\infty} (dx/dt) \left(\lim_{S \to 0} e^{-St}\, dt \right) = S\,X(S) - x(0)$$

$$= \int_0^{\infty} (dx/dt)\, dt = \left[x(t) \right]_0^{\infty}$$

$$\lim_{S \to 0} S\,X(S) - \cancel{x(0)} = x(\infty) - x(0)$$

$$x(\infty) = \lim_{S \to 0} S\,X(S)$$

Inverse Laplace Transformation:

The inverse Laplace Transform of X(S) is defined by an integral operation with respect to Variable 'S' as follows

$$x(t) = 1/2\pi \int_{-\infty}^{+\infty} X(S)\, e^{St}\, ds \quad \ldots\ldots\ldots\ldots\ldots(1)$$

Since 'S' is a complex quantity the solution requires a knowledge of complex variables.In Other words the evaluation of integral in equation (1) requires the use of contour integration In the complex plane, which is very difficult.Hence we will avoid using equation(1) to compute Inverse Laplace transform.We go for indirect methods to get the inverse Laplace transform of The given function,which are

Partial – Fraction method

Convolution integral method

Partial – Fraction method:

In many situations,the Laplace transform can be expressed in the form

$$X(S) = \frac{P(S)}{Q(S)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Where $P(S) = b_m S^m + b_{m-1} S^{m-1} + b_{m-2} S^{m-2} \ldots\ldots\ldots\ldots\ldots\ldots + b_0$

$Q(S) = a_n S^n + a_{n-1} S^{n-1} + a_{n-2} S^{n-2} + \ldots\ldots\ldots\ldots\ldots\ldots + a_0$

The function X(S)as defined by equation(2) is said to be rational function of 'S',since It is a ratio of two polynomials.The denominator Q(S) can be factored in to linear factors. A partial fraction expansion allows a strictly proper rational function $\frac{P(S)}{Q(S)}$ to be expressed As a factor of terms whose numerators are constants and whose denominator corresponds to Linear or a combination of linear and repeated factors.This in turn allows us to relate such terms To their corresponding inverse Laplace transform.

For performing partial fraction technique on X(S) the function X(S) has to meet the following conditions.

i) X(S) must be a proper fraction i.e. $m < n$. When X(S) is improper we can use long division

to reduce it to proper fraction.

ii) Q(S) should be in the factored form.

Convolution-integral method:

If L { x(t) } = X(S)
L { h(t) } = H(S)

Then L { x(t) * h(t) }= X(s) H(S)
Where '* ' is the convolution of two functions given by

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

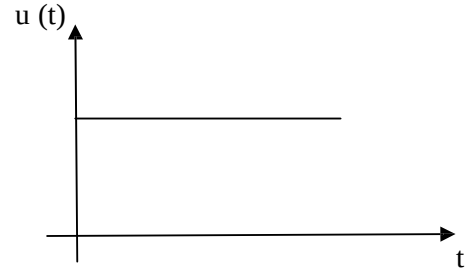
Three important Singularity functions:

The three important singularity functions employed in circuit analysis are
the unit step function u(t)
the delta function δ(t)
the ramp function r(t)

They are called Singularity functions because they are either not finite or they do not posess finite derivative everywhere.

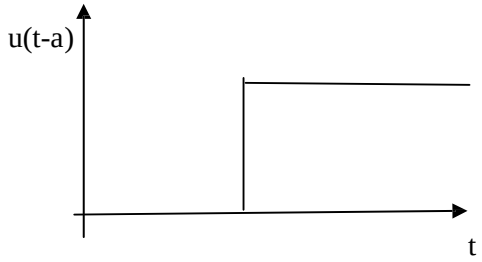
Unit step function:

The Unit-step function is defined as u(t) = 0 t< 0
= 1 t > 0



The step function can have a discontinuiy
For example in sequential switching .The unit
Step function that occurs at t=a is expressed as
u(t-a) which is expressed as

u(t-a) = 0 (t-a)< 0
= 1 (t-a)> 0



We use step function to represent an abrupt change in voltage or current , like the changes that Occur in the circuit of control engineering and digital systems.

Laplace transformation of unit step function is given by $L\{u(t)\} = \int_0 e^{-St}\,dt$

$= -1/S\,[e^{-\infty} - e^{0}]$

$= \frac{1}{S}$

Similarly $L\{u(t-a)\} = \int_a^{\infty} u(t-a)\,e^{-St}\,dt$

$= \left\{-\frac{1}{S}e^{-St}\right\}_a^{\infty} = \frac{1}{S}e^{-aS}$

Impulse function:

The derivative of the unit step function is the unit impulse function $\delta(t)$

i.e. $\delta(t) = d/dt\,\{u(t)\} = 0 \quad t < 0$

$= 1 \quad t = 0$

$= 0 \quad t > 0$

The unit impulse may be visualized as very short duration pulse of unit area This may be expressed mathematically as

$$\int_{0-}^{0+} \delta(t)\,dt = 1$$

Where t= 0- indicates the time just before t=0 and t=0+ denotes the time Just after t=0. Since the area under the unit impulse is unity, it is practice to write '1' beside the arrow. When the impulse has a strength other than unity the area of the impulse function is equal to its strength.

Since $\delta(t) = d/dt\,\{u(t)\}$

$L\{\delta(t)\} = L[d/dt\,\{u(t)\}] = S \times 1/S = 1$

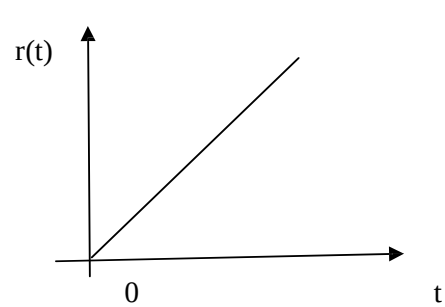
Ramp function:

Integrating the unit step function results in the unit ramp function r(t)

$r(t) = \int_{-\infty}^{t} u(\tau)\,d\tau = t\,u(t)$

$= 0 \quad t < 0$

$= t \quad t > 0$

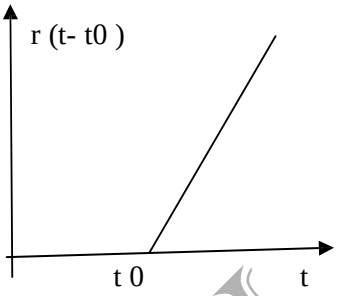


In general a ramp is a function that changes at a Constant rate.

A delayed ramp function is shown in figure Mathematically it is described as follows

$r(t-t_0) = 0 \quad t < 0$

$= t - t_0 \quad t > 0$



Laplace transformation of a ramp function is given by

$$L\{r(t)\} = L\left[\int_0^t u(t)\,dt\right]$$

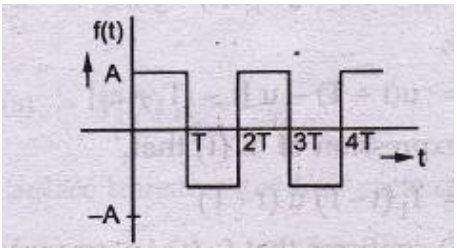
$$= 1/S \times 1/S = 1/S^2$$

$$L\{r(t-t_0)\} = 1/S \times 1/S \; e^{-t_0 S}$$

$$= 1 \; e^{-t_o S}$$

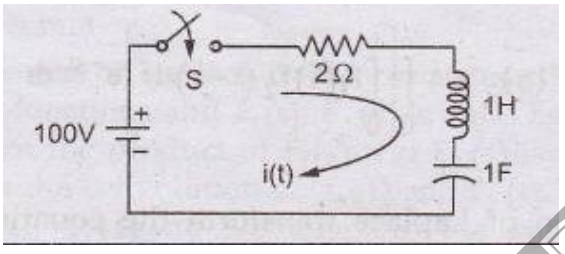
Assignment questions:

1) Obtain the Laplace transform of a periodic function with a suitable example wave form. Also find the Laplace transform of the following wave form shown in Fig.



9)) State and prove convolution theorem. Using the same find f(t) when F(s) = $\frac{1}{s^2(s+1)}$

10) Using Laplace transform determine the current in the circuit shown in Fig. 25 when the switch S is closed at t=0 . Assume zero initial condition .



11) Find the Laplace transform of i) 6(t) ii) t iii) e^{-at} iv) sin w t v)u(t)

12) Obtain the Laplace transform of sawtooth waveform in the Fig.

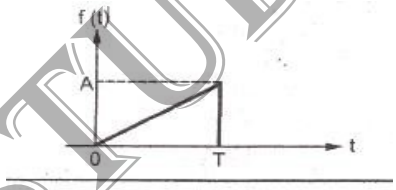


Fig. 24

13) Find the Laplace inverse of $\frac{1}{s(s+4)}$ using convolution integral

14) State and prove (i) initial value theorem and (ii) final value theorem as applied to L transform. What are the limitations of each theorem?

8) Obtain the Laplace transform of a full wave rectified sine wave of amplitude 1 and period Π secs.

9) The current function i(t) shown in fig. 10 is impressed on a capacitor C. what should be the strength A of the impulse so that the voltage across the C becomes zero for t > 5secs.

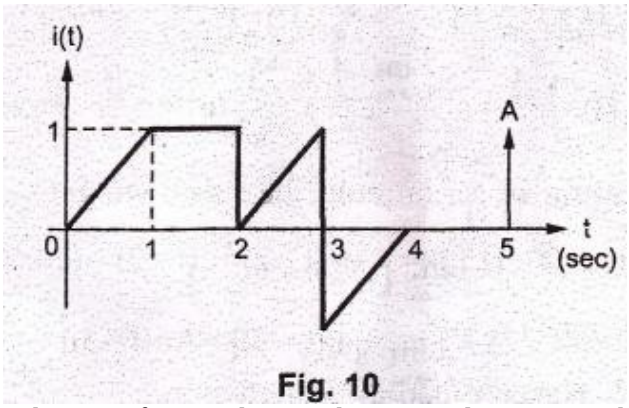


Fig. 10

10) In the circuit shown in fig. 11, the switch is opened at t = 0, with V = 1V, C = 1F L = ½ H, G = 1 find the node voltages $V_1(t)$ and $V_2(t)$ by L transform method.

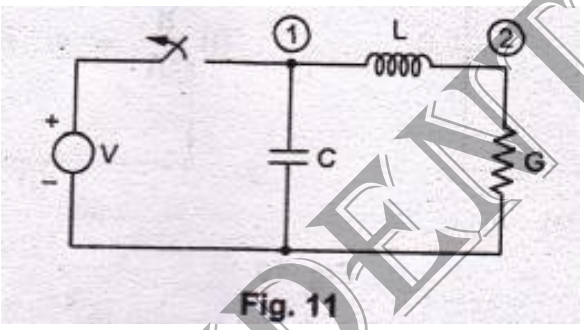


Fig. 11

11) The switch in the network of Fig. 17 open at t=0, use Laplace transformation analysis to determine the vo

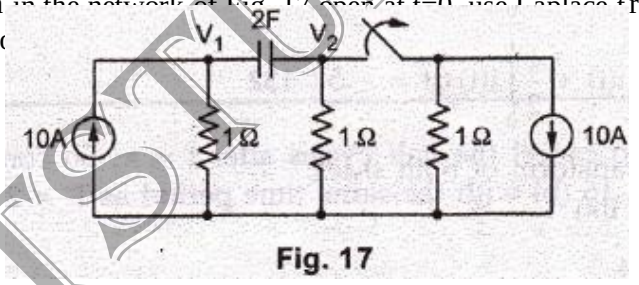


Fig. 17

Unit: 8 Two port network parameters : Hrs: 06

Syllabus of unit :

Definition of z, y, h and transmission parameters, modeling with these parameters, relationship between parameters sets.

Recommended readings:

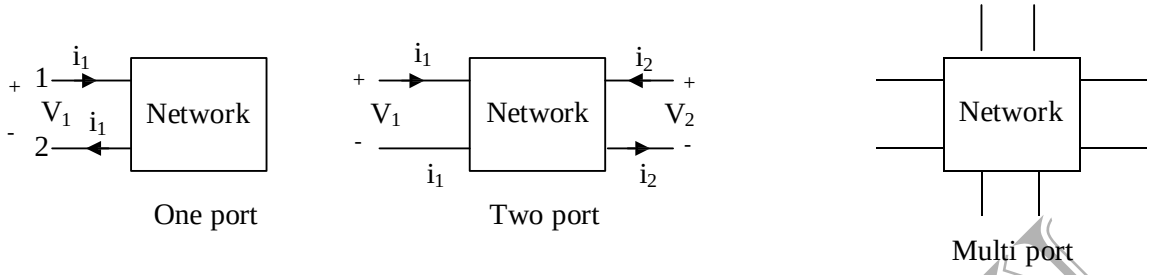
1. "Network Analysis", M. E. Van Valkenburg, PHI / Pearson Education

2. "Networks and systems", Roy Choudhury, 2 edition, New Age International Publications .

3. "Network theory ", Ganesh Rao.

4. "Network analysis" , Roy Choudry.

TWO PORT PARAMETERS:



PORT:- Pair of terminals at which an electrical signal enters or leaves a network.

One port network:- Network having only one port.

Ex: Domestic appliances, Motor, Generator, Thevinin's or Norton networks Two port network:- Network having an input port and an output port. Ex:Amplifiers,Transistors, communication circuits, Power transmission & distribution lines Filters,attenuators ,transformers etc

Multi port network:-Network having more than two ports.

Ex: PowerTransmission lines , DistributionsLines,Communication lines.

Two port networks act as building blocks of electrical or electronic circuits such as electronic systems, communication circuits, control systems and transmission & distribution systems. A one port or two port network can be connected with another two port network either in cascade, series or in parallel. In Thevinins or Nortons networks , we are not interested in the detailed working of a major part of the network. Similarly it is not necessary to know the inner working of the two port network but by measuring the voltages and currents at input and at output port, the network can be characterized with a set of parameters to predict how a two port network interact with other networks.Often the circuit between the two ports is highly complex The two port parameters provide a shorthand method for analyzing the input-output properties of two ports without having to deal directly with the highly complex circuit internal to the two port.

These networks are linear and passive and may contain controlled sources but not independent sources.inside..

While defining two port parameters we put the condition that one of the ports is either open circuited or short circuited.

In these networks there are four variables V_1, I_1 and V_2, I_2 . Two of them are expressed in terms of the other two, to define two port parameters.

Four important Parameters

Sl. No.	Parameters	Dependent Variable	Independent Variable	Equations
1.	z Parameters	V_1, V_2	I_1, I_2	$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$
2.	y parameters	I_1, I_2	V_1, V_2	$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$
3.	h parameters	V_1, I_2	I_1, V_2	$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$
4.	t parameters	V_1 , I_1	V_2 , I_2	$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$

DEFINITIONS

(1) Z parameters (open circuit impedance parameters)

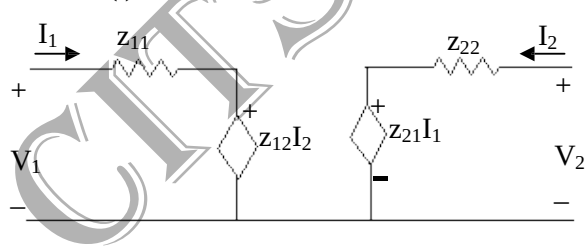
$V_1 = z_{11}I_1 + z_{12}I_2$ $\qquad z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0}$ $\qquad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0}$

$V_2 = z_{21}I_1 + z_{22}I_2$ $\qquad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0}$ $\qquad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1 = 0}$

For z_{11} and z_{21} - output port opened } Hence the name
z_{12} and z_{22} - input port opened } open circuit impedance parameters

Equivalent networks in terms of controlled sources ;

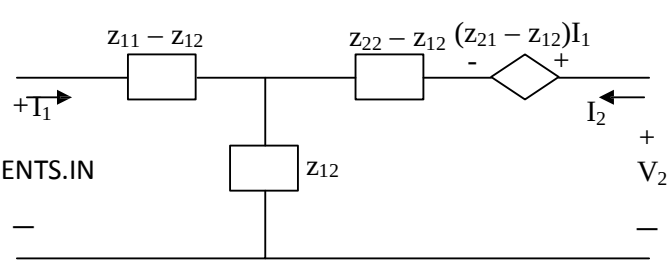
Network (i)



Network (ii) By writing

$$V_1 = (z_{11} - z_{12})\, I_1 + z_{12}\,(I_1 + I_2)$$

$$V_2 = (z_{21} - z_{12})\, I_1 + (z_{22} - z_{12})\, I_2 + z_{12}\,(I_1 + I_2)$$



The z parameters simplify the problem of obtaining the characteristics of two 2 port networks connected in series

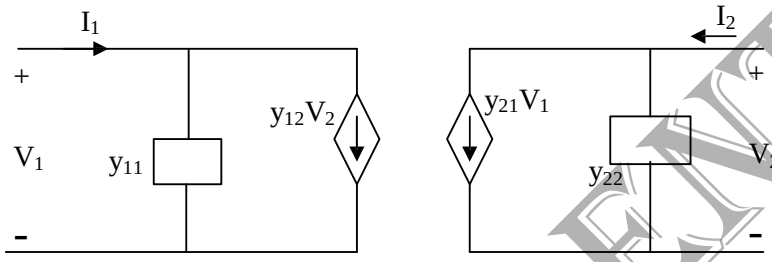
(2) y parameters

$I_1 = y_{11}V_1 + y_{12}V_2$ $\quad y_{11} = \left.\frac{I_1}{V_1}\right|_{I_2 = 0}$ $\quad y_{12} = \left.\frac{I_1}{V_2}\right|_{I_1 = 0}$

$I_2 = y_{21}V_1 + y_{22}V_2$ $\quad y_{21} = \left.\frac{I_2}{V_1}\right|_{I_2 = 0}$ $\quad y_{22} = \left.\frac{I_2}{V_2}\right|_{I_1 = 0}$

For y_{11} and y_{21} - port 2 is shorted
z_{12} and z_{22} - port 1 is shorted
} Hence they are called short circuit admittance parameters

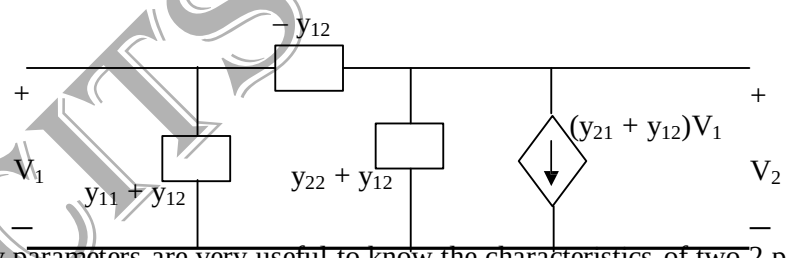
Equivalent networks in terms of controlled sources



(ii) by writing

$I_1 = (y_{11} + y_{12})\, V_1 - y_{12}\,(V_1 + V_2)$

$I_2 = (y_{21} - y_{12})\, V_1 + (y_{22} + y_{12})\, V_2 - y_{12}\,(V_2 - V_1)$



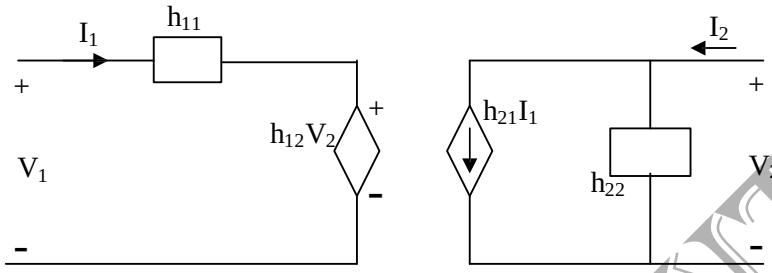
The y parameters are very useful to know the characteristics of two 2 port Networks connected in parallel

Hybrid parameters:-

$V_1 = h_{11} I_1 + h_{12} V_2$ $\quad h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2 = 0}$ $\quad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1 = 0}$

$I_2 = h_{21} I_1 + h_{22} V_2$ $\quad h_{22} = \left.\frac{I_2}{I_1}\right|_{V_2 = 0}$ $\quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1 = 0}$

Equivalent Network in terms of controlled sources;



Parameter values for bipolar junction transistors are commonly quoted
In terms of h parameters

Transmission or ABCD parameters

$V_1 = AV_2 - BI_2$ $\quad A = \left.\frac{V_1}{V_2}\right|_{I_2 = 0}$ $\quad B = \left.\frac{V_1}{-I_2}\right|_{V_2 = 0}$

$I_1 = CV_2 - DI_2$ $\quad C = \left.\frac{I_1}{V_2}\right|_{I_2 = 0}$ $\quad D = \left.\frac{I_1}{-I_2}\right|_{V_2 = 0}$

As the name indicates the major use of these parameters arise in transmission
Line analysis and when two 2 ports are connected in cascade

Relationship between two port parameters:-

Relationship between different two port parameters can be obtained as follows. From the given set of two port parameters, rearrange the equations collecting terms of dependent variables of new set of parameters to the left. Then form matrix equations and from matrix manipulations obtain the new set in terms of the given set.

(i) Relationship between z and y parameters for x parameters

$$[V] = [z][I]$$

then $$[I] = [z]^{-1}[V]$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$= \frac{1}{\Delta_z}\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad \text{where } \Delta_z = z_{11}z_{22} - z_{12}z_{21}$$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} \frac{z_{22}}{\Delta_z} & \frac{-z_{12}}{\Delta_z} \\ \frac{-z_{21}}{\Delta_z} & \frac{z_{11}}{\Delta_z} \end{bmatrix}$$

similarly $\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \frac{1}{\Delta_y}\begin{bmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{22} \end{bmatrix}$

(ii) Relationship between [y] and [h]

From $I_1 = y_{11}V_1 + y_{12}V_2$

$I_2 = y_{21}V_1 + y_{22}V_2$

Rearranging $y_{11}V_1 = I_1 - y_{12}V_2$

$y_{21}V_1 - I_2 = -y_{22}V_2$

$$\begin{bmatrix} y_{11} & 0 \\ y_{21} & -1 \end{bmatrix}\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 & -y_{12} \\ 0 & -y_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & 0 \\ y_{21} & -1 \end{bmatrix}^{-1}\begin{bmatrix} 1 & -y_{12} \\ 0 & -y_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$= \frac{-1}{y_{11}}\begin{bmatrix} -1 & 0 \\ -y_{21} & y_{11} \end{bmatrix}\begin{bmatrix} 1 & -y_{12} \\ 0 & -y_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$= \frac{-1}{y_{11}}\begin{bmatrix} -1 & y_{12} \\ -y_{21} & y_{12}y_{21} - y_{11}y_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_y}{y_{11}} \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

(iii) To Express T-parameters in terms of h-Parameters:

Equations for T-parameters,

$$\left.\begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}\right\} \longrightarrow (1)$$

Equations for h-parameters,

$$\left.\begin{aligned} V_1 &= h_{11}I_1 + h_{12}V_2 \\ I_2 &= h_{21}I_1 + h_{22}V_2 \end{aligned}\right\} \longrightarrow (2)$$

Re arranging Equation (2) $V_1 - h_{11}I_1 = h_{12}V_2$

$-h_{21}I_1 = h_{22}V_2 - I_2$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & -h_{11} \\ 0 & -h_{21} \end{bmatrix}^{-1} \begin{bmatrix} h_{12} & 0 \\ h_{22} & -1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

For which $[T] = -\dfrac{1}{h_{21}} \begin{bmatrix} -h_{21} & h_{11} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} h_{12} & 0 \\ h_{22} & -1 \end{bmatrix}$

$$= -\frac{1}{h_{21}} \begin{bmatrix} h_{11}h_{22} - h_{21}h_{12} & h_{11} \\ h_{22} & 1 \end{bmatrix} = \begin{bmatrix} -\dfrac{\Delta_h}{h_{21}} & -\dfrac{h_{11}}{h_{21}} \\ -\dfrac{h_{22}}{h_{21}} & -\dfrac{1}{h_{21}} \end{bmatrix}$$

By a similar procedure, the relationship between any two sets of parameters can be established. The following table gives such relationships:

	Y	z	H	T
[y]	$\begin{matrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{matrix}$	$\begin{matrix} \dfrac{z_{22}}{\Delta_z} & \dfrac{-z_{12}}{\Delta_z} \\ \dfrac{-z_{21}}{\Delta_z} & \dfrac{z_{11}}{\Delta_z} \end{matrix}$	$\begin{matrix} \dfrac{1}{h_{11}} & \dfrac{-h_{12}}{h_{11}} \\ \dfrac{h_{21}}{h_{11}} & \dfrac{\Delta_h}{h_{11}} \end{matrix}$	$\begin{matrix} \dfrac{D}{B} & \dfrac{-\Delta_t}{B} \\ \dfrac{-1}{B} & \dfrac{A}{B} \end{matrix}$
[z]	$\begin{matrix} \dfrac{y_{22}}{\Delta_y} & \dfrac{-y_{12}}{\Delta_y} \\ \dfrac{-y_{21}}{\Delta_y} & \dfrac{y_{11}}{\Delta_y} \end{matrix}$	$\begin{matrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{matrix}$	$\begin{matrix} \dfrac{\Delta_h}{h_{22}} & \dfrac{h_{12}}{h_{22}} \\ \dfrac{-h_{21}}{h_{22}} & \dfrac{1}{h_{22}} \end{matrix}$	$\begin{matrix} \dfrac{A}{C} & \dfrac{\Delta_t}{C} \\ \dfrac{1}{C} & \dfrac{D}{C} \end{matrix}$
[h]	$\begin{matrix} \dfrac{1}{y_{11}} & \dfrac{-y_{12}}{y_{11}} \\ \dfrac{y_{21}}{y_{11}} & \dfrac{\Delta_y}{y_{11}} \end{matrix}$	$\begin{matrix} \dfrac{\Delta_z}{z_{22}} & \dfrac{z_{12}}{z_{22}} \\ \dfrac{-z_{21}}{z_{22}} & \dfrac{1}{z_{22}} \end{matrix}$	$\begin{matrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{matrix}$	$\begin{matrix} \dfrac{B}{D} & \dfrac{\Delta_t}{D} \\ \dfrac{-1}{D} & \dfrac{C}{D} \end{matrix}$
[t]	$\begin{matrix} \dfrac{-y_{22}}{y_{21}} & \dfrac{-1}{y_{21}} \\ \dfrac{-\Delta_y}{y_{21}} & \dfrac{-y_{11}}{y_{21}} \end{matrix}$	$\begin{matrix} \dfrac{z_{11}}{z_{21}} & \dfrac{\Delta_z}{z_{21}} \\ \dfrac{1}{z_{21}} & \dfrac{z_{22}}{z_{21}} \end{matrix}$	$\begin{matrix} \dfrac{-\Delta_h}{h_{21}} & \dfrac{-h_{11}}{h_{21}} \\ \dfrac{-h_{22}}{h_{21}} & \dfrac{-1}{h_{21}} \end{matrix}$	$\begin{matrix} A & B \\ C & D \end{matrix}$

COMPUTATIONS OF TWO PORT PARAMETERS:

A. By direct method i.e. using definitions

For z parameters, open output port (I_2=0) find V_1 & V_2 in terms of I_1 by equations Calculate $Z_{11}=V_1/I_1$ &$Z_{21}=V_2/I_1$.

Open input port (I_1=0) find V_1 & V_2 in terms of I_2.Calculate $Z_{12}=V_1/I_2$ &$Z_{22}=V_2/I_2$

Similar procedure may be followed for y parameters by short circuiting the ports

h & t parameters may be obtained by a combination of the above procedures.

B. z and y parameters:By node & mesh equations in standard form

For a reciprocal network (passive without controlled sources) with only two current Sources at input and output nodes,the node equations are

$I_1=Y_{11}V_1+Y_{12}V_2+Y_13V_3+$--------- $+Y_{1n}\ V_n$

$I_2=Y_{21}V_1+Y_{22}V_2+Y_{23}V_3+$--------- $+Y_{2n}\ V_n$

$0=Y_{31}V_1+Y_{32}V_2+Y_{33}V_3+$--------- $+Y_{3n}\ V_n$

$0 = Y_{n1}V_1 + Y_{n2}\ V_2 + Y_{n3}\ V_3$---------$+Y_{nn}V_n$

then $V_1 = \frac{\Delta_{11}}{\Delta} I_1 + \frac{\Delta_{21}}{\Delta} I_2$ where Δ is the determinent of the Y matrix.

$V_2 = \frac{\Delta_{12}}{\Delta} I_1 + \frac{\Delta_{22}}{\Delta} I_2$ Δ_{1j} cofactor of Y_{1j} of Δ

Comparing these with the z parameter equations.

we have $z_{11} = \frac{\Delta_{11}}{\Delta}$ $z_{22} = \frac{\Delta_{22}}{\Delta}$ $z_{12} = \frac{\Delta_{21}}{\Delta}$ $z_{21} = \frac{\Delta_{12}}{\Delta}$

Similarly for such networks, the loop equations with voltage sources only at port 1 and 2

$$V_1 = Z_{11}I_1 + Z_{12}I_2 + \ldots\ldots\ldots + Z_{1m}I_m$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2 + \ldots\ldots\ldots + Z_{2m}I_m$$
$$O = \text{- - - - - - - - - - - - - - - -}$$
$$O = Z_{m1}I_1 + Z_{m2}I_2 + \ldots\ldots\ldots + Z_{mm}I_m$$

Then

$$I_1 = \frac{D_{11}}{D}V_1 + \frac{D_{21}}{D}V_2$$
$$I_2 = \frac{D_{12}}{D}V_1 + \frac{D_{22}}{D}V_2$$

where D is the determinant of the Z matrix and D_{ij} is the co-factor of the element Z_{ij} of Z matrix .comparing these with [y] equations

Thus we have $y_{11} = -\frac{D_{11}}{D}$ $y_{22} = -\frac{D_{22}}{D}$ $y_{12} = -\frac{D_{12}}{D}$ $y_{22} = -\frac{D_{22}}{D}$

Alternative methods

For z parameters the mesh equations are

$$V_1 = Z_{11}I_1 + Z_{12}I_2 + \ldots\ldots\ldots + Z_{1m}I_m$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2 + \ldots\ldots\ldots + Z_{2m}I_m$$
$$O = \text{----------}$$
$$O = Z_{m1}I_1 + Z_{m2}I_2 + \ldots\ldots\ldots + Z_{mm}I_m$$

By matrix partitioning the above equations can be written as

$$\begin{bmatrix} V_1 \\ V_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ Z_{31} & Z_{32} & \cdots & Z_{3n} \\ \vdots & \vdots & \vdots & \vdots \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_n \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \\ O \\ \vdots \\ O \end{bmatrix} = \begin{bmatrix} M & N \\ P & Q \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \left[M - NQ^{-1}P\right] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Similarly for Y parameters

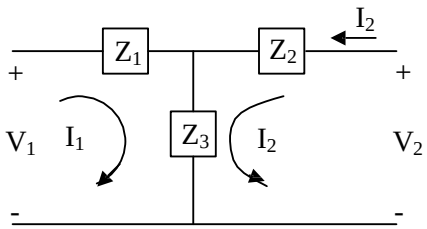
$$\begin{bmatrix} I_1 \\ I_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ Y_{31} & Y_{32} & \cdots & Y_{3n} \\ \vdots & \vdots & \vdots & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_n \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} M & N \\ O & P \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \left[M - NQ^{-1}P\right] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

C. By reducing the network (containing passive elements only) to single T or D by T-D transformations

If the network is reduced to a T network as shown



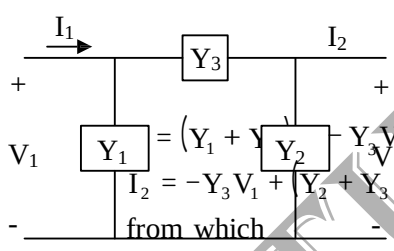
Then

$$V_1 = (Z_1 + Z_3)I_1 + Z_3 I_2$$
$$V_2 = Z_3 I_1 + (Z_2 + Z_3)I_2$$

from which

$$z_{11} = Z_1 + Z_3 \qquad z_{22} = Z_2 + Z_3$$
$$z_{12} = z_{21} = z_{13}$$

If the network is brought to IT network as shown



Then

$$I_1 = (Y_1 + Y_3)V_1 - Y_3 V_2$$
$$I_2 = -Y_3 V_1 + (Y_2 + Y_3)V_2$$

from which

$$y_{11} = Y_1 + Y_3 \qquad y_{22} = Y_2 + Y_3$$
$$y_{12} = y_{21} = -Y_3$$

SYMMETRICAL CONDITIONS

A two port is said to be symmetrical if the ports can be interchanged without changing the port voltage and currents..

$$\text{i.e.} \quad \text{if} \quad \left.\frac{V_1}{I_1}\right|_{I_2 = 0} = \left.\frac{V_2}{I_2}\right|_{I_1 = 0} \qquad \therefore \; z_{11} = z_{22}$$

By using the relationship between z and other parameters we can obtain the conditions for Symmetry in terms of other parameters.

As $z_{11}=z_{22}$, in terms of y we have $y_{11}=z_{12}/dz$ & $y_{22}=z_1/dz$, $\therefore$ $y_{11}=y_{22}$.

In terms of h parameters as $z_{11}=\Delta h/h_{22}$ & $z_{22}=1/h_{22}$ we have $\Delta h=h_{11}h_{22}-h_{12}h_{21}=1$.

In terms of t parameters as $z_1=A/C$ & $z_{22}=D/C$ the condition is A=D

Reciprocity condition in terms of two port parameters

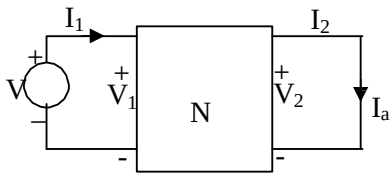


Fig 1

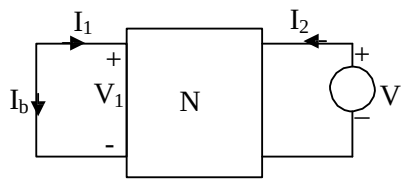


Fig 2

For the two networks shown for

Fig 1	$V_1 = V$	$I_2 = -I_a$	$V_2 = 0$
Fig 2	$V_2 = V$	$I_1 = -I_b$	$V_1 = 0$

Condition for reciprocity is $I_a = I_b$

From z parameters

$V_1 = z_{11}I_1 + z_{12}I_2$

$V_2 = z_{21}I_1 + z_{22}I_2$

we have from fig(1)

$V = z_{11}I_1 - z_{12}I_a$

$O = z_{21}I_1 - z_{22}I_a$

$$\therefore I_a = \frac{-z_{21}V}{-\Delta z} = \frac{z_{12}V}{\Delta z}$$

From fig(2)

$O = z_{11}I_b + z_{12}I_2$

$V = -z_{21}I_b + z_{12}I_2$

$$I_b = \frac{-z_{21}V}{-\Delta z}$$ then for $I_a = I_b$

$z_{12} = z_{21}$

For reciprocity with $z_{12}=z_{21}$,

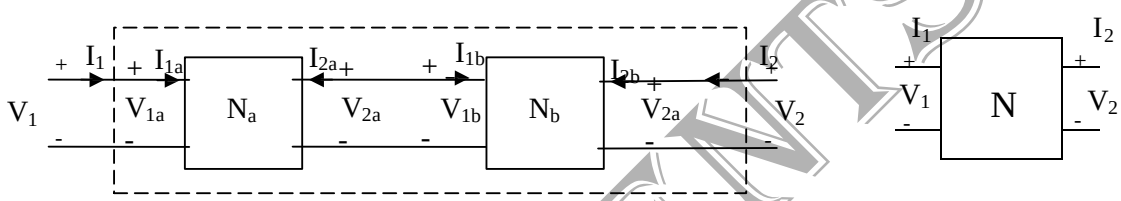
In terms of y parameters $z_{12}= - y_{12}/\Delta y$ & $z_{21}=-y_{21}/\Delta y$ condition is $y_{12}= y_{21}$

In terms of h parameters $z_{12}= h_{12}/h_{22}$ & $z_{21}= - h_{21}/h_{22}$ the condition is $h_{12}= - h_{21}$

In terms of t parameters $z_{12}=\Delta t/C$ & $z_{21}=1/C$ the condition is $\Delta t=AD - BC=1$

Parameters	**Condition for**	
	Reciprocity	Symmetry
z	$z_{12}=z_{22}$	$z_{11}= z_{22}$
y	$y_{12}=y_{22}$	$y_{11}=y_{22}$
h	$h_{12}= -h_{21}$	$h_{11}.h_{22}-h_{12}.h_{21}=1$
t	AD-BC=1	A=D

CASCADE CONNECTION:-



In the network shown 2 two port networks are connected in cascade

For N_a, $[t] = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix}$ for N_b, $[t] = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$

For the resultant network N $[t] = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$

From the cascaded network we have

$$V_{1a} = A_a V_{2a} - B_a I_{2a}$$
$$I_{1a} = C_a V_{2a} - D_a I_{2a}$$
for network N_a

$$V_{1b} = A_b V_{2b} - B_b I_{2b}$$
$$I_{1b} = C_b V_{2b} - D_b I_{2b}$$
for network N_b

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$
for network N

From the network

$I_1 = I_{1a} \quad I_{2a} = -I_{1b} \qquad I_2 = I_{2b}$

$V_1 = V_{1a} \qquad V_{2a} = V_{1b} \qquad V_2 = V_{2b}$

$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} -V_{2a} \\ -I_{2a} \end{bmatrix} \text{ and } \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

or $$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

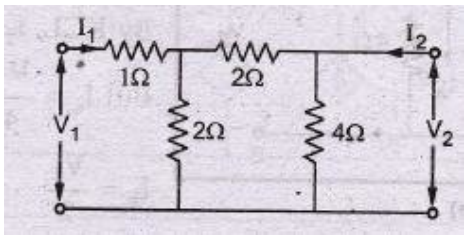
$$= \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$$

$$[T] = [T_a][T_b]$$

Assignment questions:

1) Explain Z and Y parameters with equivalent circuit Also express Z- parameters in terms of Y – parameters.
2) Find the h- parameters of the network shown in Fig. Give its equivalent circuit



3) Find Y parameters for the network shown in fig.

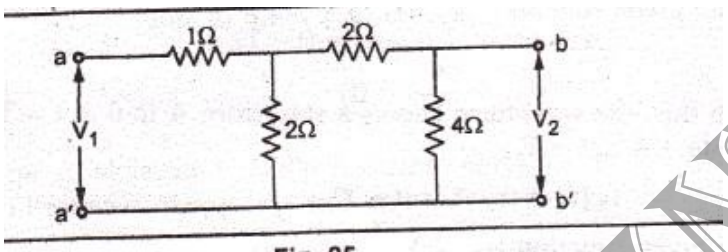


Fig. 25

4) Find the transmission or general parameters for the circuit shown in Fig

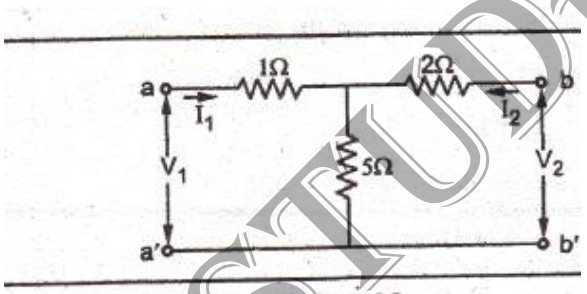


Fig. 26

5) Define y and z parameters. Derive relationship such that y parameters expressed in terms of z parameters and z parameters expressed in terms of y parameters

6) Define h and T parameters and derive expressions for [h] in terms of [T].

7) Find $[Z]$ and $[y]$ for the two port network shown in fig. 13.

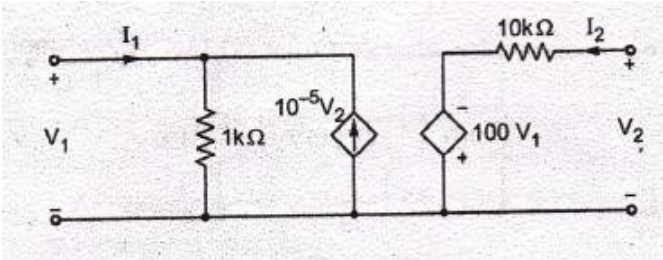


Fig. 13

8 For the network shown in fig.14 obtain the o.c. impedance parameters

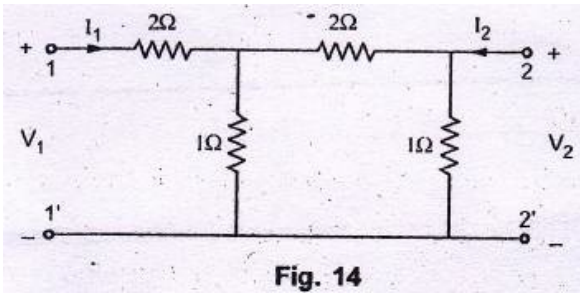


Fig. 14

9 Find $[Z]$ and $[y]$ for the two port network shown in fig. 13

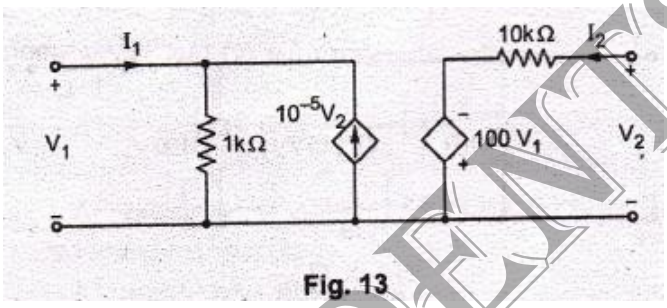


Fig. 13